

IPOs, acquisitions, and the use of convertible securities in venture capital[☆]

Thomas Hellmann*

Sauder School of Business, University of British Columbia, Vancouver, BC, Canada V6T 1Z2

Received 3 February 2004; received in revised form 27 September 2004; accepted 27 June 2005

Available online 28 February 2006

Abstract

This paper provides a new explanation for the use of convertible securities in venture capital. A key property of convertible preferred equity is that it allocates different cash flow rights, depending on whether exit occurs by acquisition or IPO. The paper builds a model with double moral hazard, where both the entrepreneur and the venture capitalist provide value-adding effort. The optimal contract gives the venture capitalist more cash flow rights in acquisitions than IPOs. This explains the use of convertible preferred equity, including automatic conversion at IPO. Contingent control rights are also important for achieving efficient exit decisions.

© 2006 Elsevier B.V. All rights reserved.

JEL classification: G24; G32; M13

Keywords: Venture capital; Convertible securities; IPO; Acquisition

1. Introduction

This paper provides a joint explanation for two central questions in venture capital contracting. First, how are so-called “exit” decisions made, where the venture either is

[☆]I would like to thank an anonymous referee, Marco Da Rin, Peter DeMarzo, Leonardo Felli, Bengt Holmström, Michael Klausner, Manju Puri, Klaus Schmidt, Gordon Smith, Lucy White, and seminar participants at Berkeley, Carnegie-Mellon, Duke, London School of Economics, New York University, Stanford University, Stockholm School of Economics, University of Amsterdam, University of New South Wales, University of Pompeu Fabra, and Yale University for their valuable comments. All errors are mine.

*Tel.: +604 822 8476; fax: +604 822 8477.

E-mail address: thomas.hellmann@sauder.ubc.ca.

acquired or remains independent to go public? Second, what explains the salient features of venture capital contracts, such as the use of convertible preferred equity and contingent control rights?

When we think about successful venture capital investments, we typically think of companies that went public, such as Apple, Intel and Yahoo. Yet acquired companies can be successful too, the most spectacular being a little-known company called Cerent, which was acquired by Cisco for \$6.9 billion. According to the National Venture Capital Association, there were more exits by acquisition than by IPO in seven out of the last 9 years (1996–2004). Most theories of venture capital financing only consider IPOs. In this paper we examine the decision between an acquisition and an IPO, and how optimal contracts can guide this decision.

In terms of the securities used in venture capital, [Sahlman \(1990\)](#), [Gompers \(1997\)](#), and [Kaplan and Strömberg \(2001, 2003, 2004\)](#) provide detailed evidence on the extensive use of convertible securities. While this has piqued the interest of a number of recent theory papers (see discussion below), the literature seems to have missed some important details of how convertible securities are actually used in venture capital. I argue that the key property of convertible preferred equity is not that it looks like debt before conversion (it needn't!), but that it allocates different cash flow rights depending on whether exit occurs through an acquisition or an IPO.

Let us briefly discuss three features of convertible securities that are easily missed. First, convertible preferred equity automatically converts in case of an IPO. The subtlety of automatic conversion is that it allocates different cash flow rights to different exit decisions: in case of an acquisition, investors own a preferred instrument, but in case of an IPO they simply own common stock. Second, the literature only examines one particular type of convertible preferred equity, where the investor can choose between a debt-like claim (without conversion the investor has a right to a preferred dividend and possibly multiple liquidation preferences) or an equity claim (with conversion the investor has rights for ordinary dividends). [Fig. 1](#) shows the return pattern for this security, which we call simple convertible preferred equity. There are several other types of convertible security, the most common being participating convertible preferred equity. [Kaplan and Strömberg \(2003\)](#) find that 40.5% of their sample uses participating convertible preferred equity, and [Fenwick and West \(2003, 2004\)](#) report that, on average, 70% of Silicon Valley deals use it. Prior to conversion, the investor holds both a debt-like and an equity claim. After conversion the investor continues to hold the equity claim, but loses the entire debt-like claim ([Fig. 2](#)). The curious aspect of this security is that an investor would never want to convert, since the rights to a preferred dividend simply vanish. This shows that the important feature of convertible preferred equity is not that it looks like debt on the downside (which is what the existing literature has focused on), but that there is automatic conversion in case of an IPO and not in an acquisition. Third, [Kaplan and Strömberg](#) find extensive use of contingent control rights in venture capital contracts. People often think that the conversion of a convertible security by itself changes control rights. But convertible preferred equity votes on an “as-if-converted” basis, so that the conversion itself makes no difference to voting rights. Yet [Kaplan and Strömberg](#) emphasize that venture capital contracts make extensive use of truly contingent control rights. We therefore need a theory that *separately* accounts for these contingent control rights.

The starting point of this paper is that the exit decision determines who continues to manage the venture. If the venture remains independent, the entrepreneur and the venture

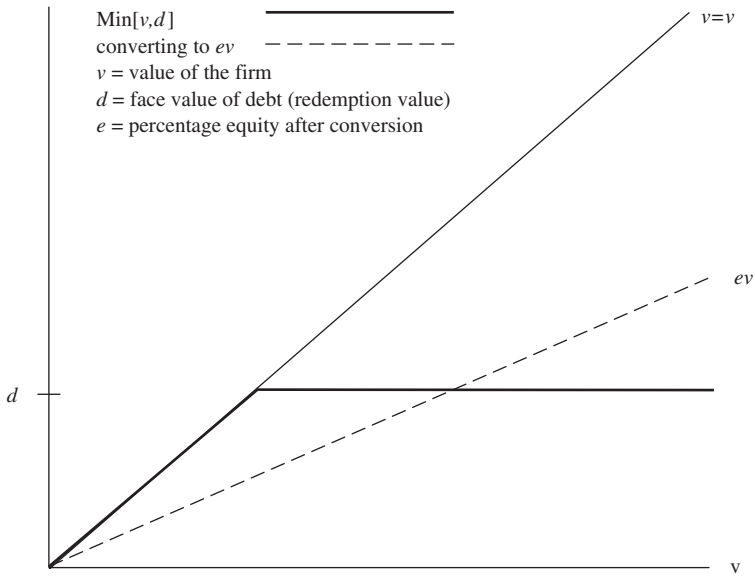


Fig. 1. Simple convertible preferred equity.

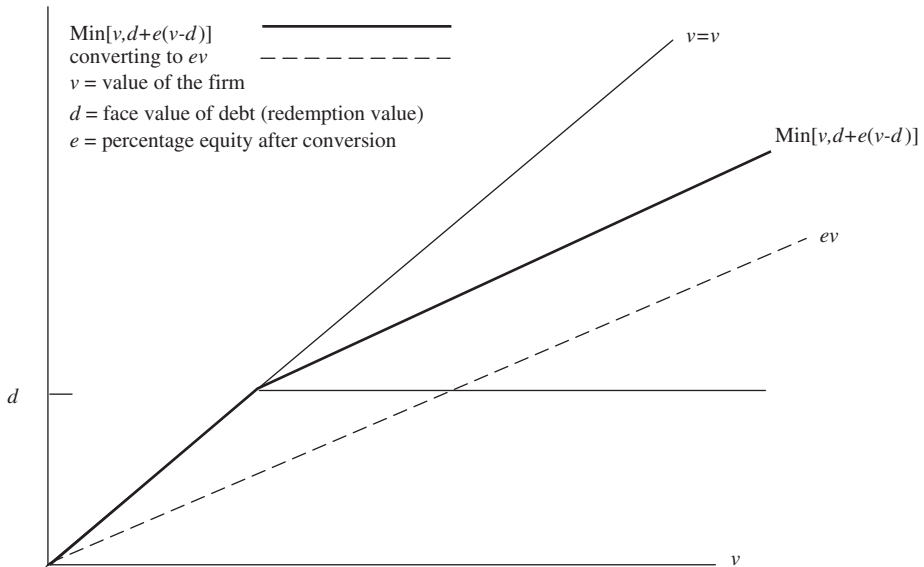


Fig. 2. Participating convertible preferred equity.

capitalist remain in charge. If it is acquired, managers of the acquiring company take over. Much of the recent venture capital literature emphasizes the fact that venture capitalists provide value-adding support (Sahlman, 1990; Hellmann and Puri, 2000, 2002). I model the value-adding process as a double moral hazard, where both the entrepreneur and the venture capitalist provide effort (Inderst and Müller, 2004). If the venture remains

independent, the optimal security is pure equity. This explains the use of automatic conversion clauses.

If the venture is acquired, a similar value-adding process exists, but it is now in the acquiring company's hand. The entrepreneur and the venture capitalist only care about their respective stakes in the acquisition value. This implies an important asymmetry. If the venture remains independent, the allocation of cash flow rights has an ongoing incentive effect. But if the venture is acquired, the division of the acquisition value merely redistributes utility between the entrepreneur and the venture capitalist. If the venture has a low funding requirement, pure equity works fine. But for a higher funding requirement, the venture capitalist requires additional cash flow rights to break even. This is where convertible preferred equity becomes optimal. If the venture goes public, the automatic conversion to equity ensures efficient incentives. And the preferred terms prior to conversion allow the venture capitalist to extract additional cash flows in case of acquisition. Note that for a given value realization, the optimal contract could require that the venture capitalist receive a higher return if the value is realized through an acquisition rather than an IPO. This cannot be done with a simple combination of debt and equity. But convertible preferred equity, with its automatic conversion in case of IPO, naturally implements this optimal contract.

Once we establish the optimality of convertible preferred equity, we are faced with the challenge of implementing efficient exit decisions. The model allows for a general two-dimensional distribution of acquisition prices and expected IPO values. The problem is that convertible preferred equity biases exit preferences. For lower IPO values, the entrepreneur wants to remain independent, even when an acquisition is more efficient. And for higher IPO values, the venture capitalist might still want to do the acquisition, even if an IPO would be more efficient. I solve the ensuing renegotiation game. Giving the entrepreneur control works well for lower funding levels. But for a larger funding requirement, the venture capitalist has to make too many concessions at the renegotiation stage. Giving full control to the venture capitalist is not optimal either, because the entrepreneur has a binding wealth constraint, which hampers renegotiation and can lead to inefficient exit decisions. For a larger funding requirement, the optimal contract therefore requires a contingent control structure. Control shifts from the venture capitalist to the entrepreneur whenever the venture has sufficiently good prospects for going public. This can be implemented using milestones like those described by Kaplan and Strömberg.

Venture capital contracts contain a high level of complexity. My theoretical model hopes to provide a framework for understanding their overall logic. One limitation is that my model can only provide a single rationale for each contractual clause, when in reality there may be multiple rationales. Moreover, it cannot explain all the clauses that are empirically observed. I provide a series of model extensions to explore at least some of these other contractual clauses. Among other things, I explore when simple versus participating convertible preferred equity is optimal; why an optimal contract can have further nonlinearities; and how to structure optimal milestones.

The paper contributes to the literature on convertible securities. Seminal papers on this topic include [Green \(1984\)](#) and [Stein \(1992\)](#). More recently, some theory papers focus specifically on the use of convertible securities in venture capital contracts. [Schmidt \(2003\)](#) examines a sequential investment problem where the entrepreneur completes her private investment before the investor makes his. The convertible security provides an optimal incentive mechanism that ensures that the investor converts if and only if the entrepreneur

invested efficiently. Cornelli and Yosha (2003) examine the incentives of the entrepreneur for short-term window dressing. They show how a convertible security might alleviate this problem. Berglöf (1994) examines an incomplete contract model where a monopolistic acquirer could take advantage of the investor by bargaining only with the entrepreneur. Under some assumptions about the distribution of private benefits, convertible debt protects the investor from asset stripping, and protects the entrepreneur from a loss of private benefits. Other related models include Bascha and Walz (2001), Bergemann and Hege (1998), Casamatta (2003), Cestone (2001), Cestone and White (2003), Dessi (2005), Hellmann (2003), Marx (1998), and Repullo and Suarez (2004). In many of these models, however, the convertible security is indistinguishable from a simple combination of debt and equity. Berglöf (1994) and Bascha and Walz (2001) are the only ones to recognize the trade-off between acquisitions and IPOs. Hellmann (2003) and Schmidt (2003) are the only ones to include a discussion of automatic conversion. This paper explicitly shows why a convertible security is needed, why a simple combination of debt and equity is not enough, and why the convertible security should convert in case of an IPO.

The analysis of control rights builds on the work of Aghion and Bolton (1992) and Hart and Moore (1998). They use a model of incomplete contracts with private benefits to explain the optimal allocation of contingent control rights. The current model does not rely on private benefits to derive a conflict of interest between the entrepreneur and investor, but derives a conflict endogenously. In a related vein, Hellmann (1998) examines the importance of giving the venture capitalist control rights for the replacement of founders. Dessein (2005) examines the preference congruence between an entrepreneur and a venture capitalist, and how it affects the willingness of an entrepreneur to relinquish control. Kirilenko (2001) models control rights as private benefits. Aghion et al. (2004) examine how liquidity shocks affect a venture capitalist's desire to exit an investment. The analysis also builds on Black and Gilson's (1997) verbal argument that venture capital contracts include a pre-commitment to return control rights to the entrepreneur at the time of an IPO. Finally, Gilson and Schizer (2003) show that from a tax perspective, convertible preferred equity has several advantages. But they are careful to point out that taxes alone cannot explain the use of convertible preferred equity, because there are many other securities that can also achieve similar tax advantages.

The remainder of the paper is structured as follows. Since the full model involves some complexity, Section 2 first examines a simplified model that explains the key insights. Section 3 develops the full model. Section 4 shows how the model can be used to explain additional features of venture capital contracts. It is followed by a brief conclusion. All proofs are in the Appendix.

2. The simplified model

2.1. Base assumptions

Suppose there is an entrepreneur, denoted by E , who wants to start a new venture requiring an investment k . E 's wealth is limited by w , so that the funding requirement is $x = k - w$. E seeks funding from a set of perfectly competitive venture capitalists, denoted by V . All parties are risk-neutral and there is no discounting.

There are three dates, denoted by 0, 1, and 2. Fig. 3 shows the time line. At date 0, the entrepreneur and the venture capitalist write an optimal contract, discussed below. At date

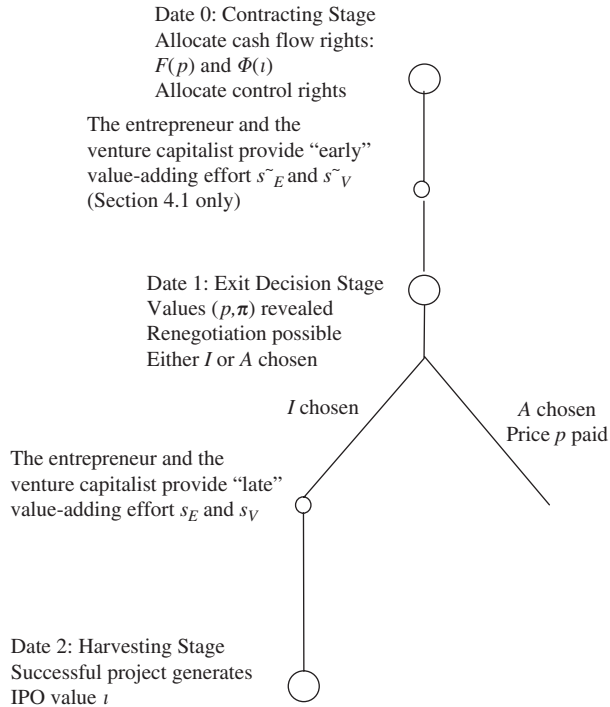


Fig. 3. Timing of the game.

1, the company is either a failure, and we assume for simplicity that there is a zero liquidation value, or, with probability $\omega (\leq 1)$, the company remains viable. In this case, a decision has to be made. The venture can be sold to an acquirer; we denote this decision by A , and the competitive acquisition price by p . Or the venture can remain independent, hoping to go public at date 2; we denote this decision by I . Remaining independent is risky. In case of failure there are no returns. The probability of success is given by σ (in the full model I explain this in greater detail). In case of success, the IPO value for existing shareholders (also called the pre-money valuation) is denoted by i . Let us denote the expected utilities of choosing I at date 1 by u_E and u_V . Let $u = u_E + u_V$. In the simplified model, we have $u = \sigma si$. Assume that p and u are verifiable and revealed at date 1. In this section I use a highly simplified specification, where p always takes the same value, and u only takes two values with $u_L < p < u_H$. The lower value u_L stands for a smaller IPO value and/or for a lower probability of success. The terms lower and higher should be thought of in relative, not absolute, terms, so that a lower IPO value can still represent an attractive return. The main difference is that in the higher state, I is the efficient choice, whereas in the lower state, A is the efficient choice. Let ω_H (ω_L) be the ex ante probability of the higher (lower) state, so that $\omega_H + \omega_L = \omega (\leq 1)$.

2.2. Contracts

At date 0, the entrepreneur and the venture capitalist design an optimal contract that satisfies the venture capitalist's zero profit condition. The contract allocates cash flow and

control rights. In this model, control rights pertain to the exit decision. I distinguish between entrepreneur control, venture capital control, and contingent control, where the allocation of control rights depends on the realization (p, u) . Prior to the exit decision, there can be renegotiation. For simplicity assume that the venture capitalist has all the bargaining power in case of renegotiation (The Appendix shows that this assumption hardly matters at all).

Cash flow claims can be defined separately for A and I . Denote the venture capitalist's claims by $F(p)$ in case of A , and $\Phi(i)$ in case of I . I identify some common securities that play an important role in the analysis. Debt corresponds to $F(p) = \text{Min}[p, d]$ and $\Phi(i) = \text{Min}[i, d]$, where d is the face value of debt. Equity corresponds to $F(p) = ep$ and $\Phi(i) = ei$, where e is the venture capitalist's percentage equity stake. Simple convertible preferred equity, with automatic conversion in case of IPO, corresponds to $F(p) = p$ for $p \leq d$ and $F(p) = \text{Max}[d, ep]$ for $p > d$, and $\Phi(i) = ei$.¹ Section 4.1 discusses other types of convertible preferred equity.

2.3. Incentives

If the venture is sold, the entrepreneur and venture capitalist retreat from the venture. But if they choose to remain independent, they continue to play an important productive role. The venture capital literature has emphasized the importance of incentives and conceptualized this as a double moral hazard problem (see Hellmann, 1998, 2002; Inderst and Müller, 2004; Repullo and Suarez, 2004). If both the entrepreneur and venture capitalist contribute value, they both should have equity-like claims that allow them to participate in the upside. I focus on the double moral hazard after date 1 (Section 4.1 discusses additional moral hazard problems before date 1), so that the probability of success depends on the private effort of the entrepreneur and venture capitalist. The full model provides a complete specification of the double moral hazard problem. This section uses the following drastic simplification: Let e^* denote the optimal equity stake of the venture capitalist. Let us assume that any other division of equity generates a much lower probability of success. Moreover, for now let us ignore private effort costs.

2.4. Optimal cash flow rights

Consider pure equity with $e = e^*$. If A is chosen in the lower state and I in the higher state, the ex ante utilities of the venture capitalist and entrepreneur are given by

$$U_E = \omega_H(1 - e^*)u_H + \omega_L(1 - e^*)p - W \quad \text{and} \quad U_V = \omega_H e^* u_H + \omega_L e^* p - X, \quad (1)$$

where W and X denote the entrepreneur's and venture capitalist's actual investment. For $X > x$ the entrepreneur reduces the financial investment to $W = w - (X - x)$. Perfect competition requires $U_V = 0$. We immediately obtain $X = \omega_H e^* u_H + \omega_L e^* p \equiv x_1$.

Having the venture capitalist invest $X = x_1$ in return for an equity stake e^* works well as long as the entrepreneur has a moderate funding need, i.e., as long as $x \leq x_1$. For $x > x_1$,

¹It is worth noting that for a typical convertible security, the debt (or redemption) value d is affected by two separate cash flow claims. First, preferred shares are entitled to a preferred dividend. Let r be the dividend rate and δ be the cumulative percentage dividend claim. Typically dividends accrue but do not compound, so that $\delta = rt$ (with compounding, it would be $\delta = (1 + r)^t - 1$). Second, venture capital contracts can specify multiple liquidation preferences, which I denote by λ . The total debt value is thus given by $d = \lambda X + \delta X$.

however, the venture needs more external funding. The venture capitalist is unable to break even with the above equity contract. It would be inefficient to increase the venture capitalist's equity stake above e^* , since this would undermine the entrepreneur's incentives. This is where convertible preferred equity comes in. It increases the venture capitalist's stake only in case of an acquisition. The expected returns with convertible preferred equity are given by

$$U_E = \omega_H(1 - e^*)u_H + \omega_L(p - d) - w \quad \text{and} \quad U_V = \omega_H e^* u_H + \omega_L d - x. \quad (2)$$

Competition determines the optimal level of d , i.e., $U_V = 0 \Leftrightarrow d = (x - \omega_H e^* u_H) / \omega_L$. Convertible preferred equity provides a simple solution to the fundamental problem of providing the venture capitalist a sufficient ex ante return, while preserving the balance of incentives in case the firm attempts to go public. Convertible preferred equity is optimal whenever the entrepreneur has a larger funding requirement ($x > x_1$).

2.5. Optimal control rights

Using convertible preferred equity solves one problem, but it can create other problems. The assumption so far is that I is chosen in the higher state and A in the lower state. Let us now examine how to ensure this, beginning with the case where the entrepreneur has control. In the lower state, the entrepreneur is tempted to choose I instead of A whenever $(1 - e^*)u_L > p - d$. This is inefficient, so the venture capitalist would want to renegotiate. He can propose to lower his stake to $d^R = p - (1 - e^*)u_L$, which is just enough for the entrepreneur to agree to an acquisition. The venture capitalist benefits from this renegotiation since $d^R > e^* u_L$ (which follows from $p > u_L$). However, once we take this renegotiation into account, we realize that setting d above d^R is pointless. This puts a limit on how much cash flow the venture capitalist can extract, and thus how much external financing is feasible. Formally, giving the entrepreneur control works well for $U_V = \omega_H e^* u_H + \omega_L d^R - x \geq 0 \Leftrightarrow x \leq x_2 \equiv \omega_H e^* u_H + \omega_L(p - (1 - e^*)u_L)$. Note that $x_1 < x_2$, so that using convertible preferred equity and giving the entrepreneur control extends the range of ventures that can be financed.

Now consider giving control to the venture capitalist. This solves the problem in the lower state: since $d > e^* p > e^* u_L$, the venture capitalist always prefers A over I . But venture capital control can create a problem in the higher state, because the venture capitalist prefers A over I whenever $d > e^* u_H$.² Under venture capital control, however, renegotiation is more difficult. Since the entrepreneur has no wealth, the only thing the venture capitalist can ask for is a higher equity stake, raising e above e^* . But this upsets the balance of incentives. In the full model, I show that two things can happen: either renegotiation is possible but leads to unbalanced incentives; or renegotiation is impossible altogether (see also [Aghion and Bolton, 1992](#)). This section focuses on the latter case. For $d > e^* u_H$, there are some inefficient exits, where A is chosen even though I is efficient. The expected utilities are simply given by $U_V = \omega d - x$ and $U_E = \omega(p - d) - w$. The feasible range under venture capital control is given by $x \leq \omega p \equiv x_4$, which is always larger than the feasible range under entrepreneur control, i.e., $x_2 < x_4$.³

²I focus here on the case where $p > e^* u_H$. For $p < e^* u_H$ there is no problem with venture capital control. Note that in the full model, there always exists a range of p 's that correspond to $p > e^* u_H$.

³The paper assumes throughout that the entrepreneur's participation constraint is always satisfied. This can be justified by arguing that she receives a private benefit from starting her own venture. Alternatively, we can relax

We have not yet exploited all contracting possibilities. In particular, control rights can also be made contingent on new information at date 1. It is easy to see how this additional contractual flexibility is valuable. We just saw that giving the entrepreneur control causes problems in the lower state, and giving the venture capitalist control causes problems in the higher state. Contingent control solves this problem by giving control to the entrepreneur in the higher state and to the venture capitalist in the lower state. The entrepreneur always wants to choose I in the higher state, and the venture capitalist always wants to choose A in the lower state. With contingent control, the feasible range is found as follows: $U_V = \omega_H e^* u_H + \omega_L p - x \geq 0 \Leftrightarrow x \leq x_3 \equiv \omega_H e^* u_H + \omega_L p$. Note that $x_2 < x_3$ so that contingent contracts extend the feasible range relative to entrepreneur control. And comparing contingent control to venture capital control, we note that $x_3 < x_4$: while contingent control cannot extend the feasible range, it can extend the range where efficient contracting is possible.

3. The full model

The simplified model is useful for laying out the logical flow of the argument and presenting many of the key insights. The results of the full model match those of the simplified model, and provide some additional insight. Let us briefly review why we even need the full model. First, we “cheated” with the simplified specification of the double moral hazard problem. Second, we severely restricted the parameter space of p and u . Third, while the simplified model shows that convertible preferred equity is an optimal security, it cannot explain why a combination of debt and equity might be insufficient.

We have already stated that the probability of an IPO is given by σ . Let us now note that σ is an exogenous scalar on the probability of success, and s is endogenously determined by the efforts of the entrepreneur and venture capitalist. It is useful to define $\pi = \sigma i$, where π provides an index for how attractive it is to choose I . At date 1 all parties observe p and π .⁴ The realizations of p and π are likely to be correlated, since a better company is likely to attract higher values for both. The model can allow for any bivariate distribution of p and π . Denote this distribution by $\Omega(p, \pi)$ and its density by $\omega(p, \pi)$. For expositional simplicity let us assume that $\Omega(p, \pi)$ has a support $[0, \bar{p}] \times [0, \bar{\pi}]$, where $0 < \bar{p} < \infty$ and $0 < \bar{\pi} < \infty$.⁵ Cash flow claims are again defined separately for A and I . Denote the venture capitalist's claims by $F(p)$ in case of A , and $\Phi(i)$ in case of I . Cash flow and control rights can be made contingent in the realization of (p, π) . The model thus allows for complete contracting on all observable information.⁶

(footnote continued)

this assumption and incorporate a binding participation constraint. It is easy to verify that the feasible range is given by $x \leq \omega p - w$.

⁴The model continues to hold if instead of observing i , everybody only observes a signal ι of i ; in this case, π is given by $\pi = \sigma E(i|\iota)$, where $E(i|\iota)$ is the expected value of i , given the signal ι .

⁵Venture capital contracts do not make a legal distinction between liquidations and acquisitions. We can therefore subsume liquidations under acquisitions. The distribution $\Omega(p, \pi)$ is likely to have a mass point at (or close to) $p = \pi = 0$, representing the case where the venture is a total failure at date 1.

⁶We specify the optimal contract in terms of cash flow rights and control rights over the exit decision. Since contracts are complete, it is possible to reinterpret control rights in terms of contractually pre-specified decisions over A versus I . In practice, it would be very difficult to pre-specify exit decisions, so that venture capital contracts always allocate control rights over exit decisions. The reason for this is presumably that there are additional actions, not formally modeled here, that are necessary for implementing exit decisions (e.g., finding and

3.1. Incentives

Our specification of the double moral hazard problem is similar to Inderst and Müller (2004). If I is chosen at date 1, the entrepreneur and the venture capitalist continue to own and manage the venture.⁷ The probability of success is given by σs with $s = \theta_E s_E + \theta_V s_V$, where s_E and s_V are the respective private efforts. For simplicity, let us use quadratic effort costs of $s_E^2/2$ and $s_V^2/2$. The coefficients θ_E and θ_V measure respective productivities. The Appendix verifies that the results are also robust in a Cobb–Douglas model, where there are complementarities between private effort. This is also consistent with Inderst and Müller (2004), who explain that a variety of double moral hazard specifications yield similar results.

It is natural to express the venture capitalist's cash flow rights as an equity stake. That is, $\Phi(i) = ei$, where e represents the venture capitalist's percentage equity stake. Below I show that the optimal e is a constant.

Let u_E and u_V denote the respective expected utilities in case of I , given by

$$u_E = (\theta_E s_E + \theta_V s_V)(1 - e)\pi - \frac{s_E^2}{2} \quad \text{and} \quad u_V = (\theta_E s_E + \theta_V s_V)e\pi - \frac{s_V^2}{2}. \quad (3)$$

The entrepreneur chooses s_E to maximize u_E . From the first-order condition we obtain $s_E = \theta_E(1 - e)\pi$. The venture capitalist chooses s_V to maximize u_V , and we obtain $s_V = \theta_V e\pi$. Increasing e increases the venture capitalist's effort, but reduces the entrepreneur's effort.

A standard result in double moral hazard models is that it is impossible to achieve a “first-best efficient” outcome (this would be $s_V = \theta_V \pi$ and $s_E = \theta_E \pi$). However, it is possible to identify a “constrained efficient” outcome that maximizes the entrepreneur's and venture capitalist's joint utility, given their respective incentive constraints. For the remainder of the paper the term “efficient” refers to this constrained efficient outcome. Formally, the efficient share e^* maximizes $u(e) = u_V(e) + u_E(e)$, subject to $s_V = \theta_V e\pi$ and $s_E = \theta_E(1 - e)\pi$.

Proposition 1. *The optimal security in case of I is always pure equity. The efficient allocation level of equity is given by $e^* = \theta_V^2 / (\theta_E^2 + \theta_V^2)$. For $e < e^*$ the venture capitalist has too few incentives and for $e > e^*$ the entrepreneur has too few incentives.*

Fig. 4 shows the utility frontier and identifies the efficient outcome. For $e < e^*$, the venture capitalist's stake is too small. But he can always solve this problem by acquiring an additional stake: hence the linear portion of the utility frontier. For $e > e^*$, the entrepreneur's stake is too small. Since the entrepreneur is wealth-constrained, she cannot change this inefficiency: hence the curved portion of the utility frontier.

(footnote continued)

bargaining with an acquirer). If these actions are difficult to specify contractually, it makes more sense to allocate control rights.

⁷The model considers the value creation process in an independent company, but treats the acquisition as a black box. The working paper version shows that relaxing this assumption does not change the fundamental insights of the model. After the acquisition, the firm could well have its own value creation process and face its own internal incentive problems. The parameters of this internal process determine the value of acquiring the venture. From the perspective of the entrepreneur and venture capitalist, all that matters is the resulting acquisition price.

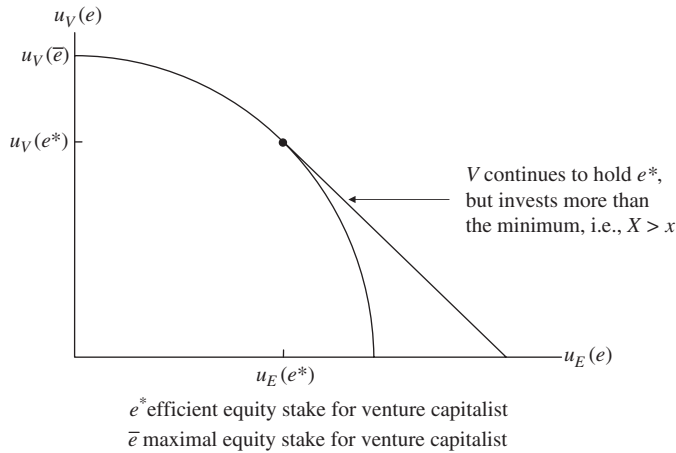


Fig. 4. Utility frontier of the entrepreneur and the venture capitalist under I .

The Appendix shows that the maximal level of equity is given by $\bar{e} = \theta_E^2 / (2\theta_E^2 - \theta_V^2)$. We have $\bar{e} < 1 \Leftrightarrow \theta_E > \theta_V$. For $\bar{e} < e \leq 1$, the venture capitalist would voluntarily give up some equity, since the benefit of providing better incentives to the entrepreneur exceeds the cost of giving up equity.

Proposition 1 shows that pure equity is optimal. Empirically, this is very close to what venture capital contracts look like, where automatic conversion of convertible preferred equity ensures that the venture capitalist holds ordinary stock in case of an IPO. Section 4.2 examines this in further detail.

For the theoretical proof, note that after date 1, we only have a two-point distribution: i in case of success and nothing in case of failure. For a given i , it might seem like there are other securities that could also give the venture capitalist his required stake. However, the key is to note that the optimal stake e^* does not depend on σ or i (and thus π). This means that from an ex ante point of view, the venture capitalist's optimal cash flow rights are given $\Phi(i) = e^*i$ for all possible realizations of σ and i . Hence, the optimal contract is linear in i , and equity is the uniquely optimal security. The key intuition is that equity maintains the optimal balance of incentives for all realizations of σ and i .

The paper focuses throughout on the model where $\theta_E \geq \theta_V$. This is the more plausible model: almost by the definition of the job title, the entrepreneur is likely to have a bigger impact on company outcomes than the venture capitalist. Sections 3.2 and 3.3 simplify the exposition by focusing on the symmetric case of $\theta_E = \theta_V$, where $e^* = \frac{1}{2}$. The Appendix extends all proofs to the general asymmetric model, and shows that the results are the same.

3.2. Renegotiation

Let us now turn to the question of how the exit decision is made. Let the entrepreneur's and the venture capitalist's relative preference for I over A be given by $\Delta_E \equiv u_E(e, \pi) - (p - F(p))$ and $\Delta_V \equiv u_V(e, \pi) - F(p)$. The exit decision is efficient if, for a given e , it maximizes the joint utility of the entrepreneur and venture capitalist. This is equivalent to choosing I for $\Delta_E + \Delta_V > 0$ and choosing A for $\Delta_E + \Delta_V < 0$.

The exit decision itself depends on whether the entrepreneur or the venture capitalist has control. To solve the renegotiation game we derive four critical values of π . Suppose the entrepreneur has control. Prior to renegotiation, she prefers I for $\Delta_E > 0$ and A for $\Delta_E < 0$. We define $\hat{\pi}_E$ so that $\Delta_E(\hat{\pi}_E) = 0$. Prior to renegotiation, the entrepreneur prefers I for $\pi > \hat{\pi}_E$, and A for $\pi < \hat{\pi}_E$. If the venture capitalist has control, prior to renegotiation, he prefers I over A whenever $\Delta_V > 0$. The second critical value is given by $\Delta_V(\hat{\pi}_V) = 0$. The venture capitalist prefers I for $\pi > \hat{\pi}_V$ and A for $\pi < \hat{\pi}_V$.

These two critical values determine whether the controlling party wants to renegotiate. We need two additional critical values to determine whether renegotiation is feasible. Remember that the entrepreneur faces a binding wealth constraint. Without the wealth constraint, renegotiation would be simple, since the two parties could use wealth transfers to reach an efficient agreement. But with a wealth constraint, the only currency available to the entrepreneur is to give up more of her cash flow rights. This has no incentive effects if the decision is A . But if the decision is I , giving up cash flow rights reduces her incentives, which in turn reduces the value of choosing I .

The critical value $\hat{\pi}_A$ satisfies $u(e, \hat{\pi}_A) = p$. Renegotiation to A is possible (and efficient) whenever $\pi < \hat{\pi}_A$, but impossible (and inefficient) for $\pi > \hat{\pi}_A$. The Appendix shows that there also exists a critical value $\hat{\pi}_I$, so that renegotiation to I is possible whenever $\pi > \hat{\pi}_I$, but impossible for $\pi < \hat{\pi}_I$. Note that $\hat{\pi}_I$ and $\hat{\pi}_A$ are different precisely because renegotiation to I involves additional incentive issues, whereas renegotiation to A does not.

From Section 3.1 we know that the venture capitalist never holds less than his optimal share of equity. We can therefore focus on contracts with $e \geq e^* = \frac{1}{2}$. Anticipating the analysis of optimal contracts, we can also limit our attention to those contracts that give the venture capitalist the same or stronger cash flow rights in an acquisition than in an IPO, that is, contracts with $F(p) \geq ep$ for all p .

Proposition 2 (*Renegotiation outcomes*). (i) If, for a given p , $F(p) = ep$ and $e = \frac{1}{2}$, then $\hat{\pi}_V = \hat{\pi}_I = \hat{\pi}_A = \hat{\pi}_E$.

(ii) If, for a given p , $F(p) > ep$ and $e \geq \frac{1}{2}$, then $\hat{\pi}_V > \hat{\pi}_I > \hat{\pi}_A > \hat{\pi}_E$.

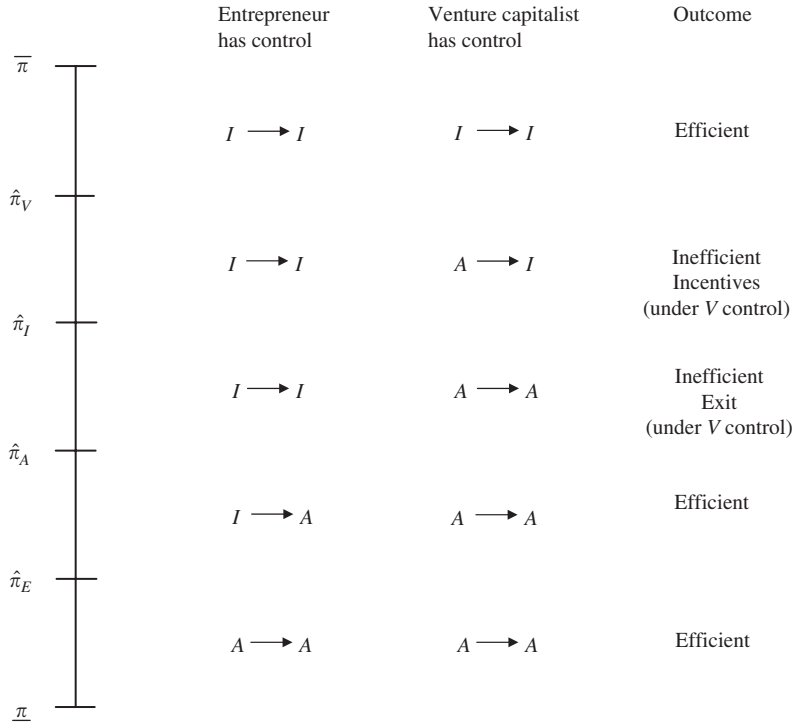
(iii) If the entrepreneur has control, I is chosen for $\pi > \hat{\pi}_A$ and A is chosen for $\pi < \hat{\pi}_A$. The exit decision is always efficient.

(iv) If the venture capitalist has control, I is chosen for $\pi > \hat{\pi}_I$, and A is chosen for $\pi < \hat{\pi}_I$. For $\hat{\pi}_A < \pi < \hat{\pi}_I$, renegotiation is impossible, so that A is chosen, even though I is efficient. And for $\hat{\pi}_I < \pi < \hat{\pi}_V$, renegotiation from A to I is possible, but decreases the entrepreneur's stake, which worsens incentives.

Fig. 5 accompanies Proposition 2. $F(p) = ep$ and $e = \frac{1}{2}$ is a benchmark case, where the entrepreneur and venture capitalist have symmetric ownership stakes. Their interests are perfectly aligned, and there is never any disagreement about the exit decision. But if the venture capitalist receives stronger cash flow rights from A , he develops a bias towards acquisitions. Let us now focus on this problem.

The case where the entrepreneur has control is relatively straightforward. For $\pi > \hat{\pi}_A$, the entrepreneur prefers I which is efficient. For $\pi < \hat{\pi}_E$, the entrepreneur prefers A which is also efficient. For $\hat{\pi}_E < \pi < \hat{\pi}_A$, the entrepreneur initially prefers I which is inefficient. It is easy to renegotiate to A . The venture capitalist makes A more attractive to the entrepreneur by lowering his stake. This ensures an efficient exit decision.

In the case of venture capital control, renegotiation might not always generate an efficient outcome. Indeed, for $\hat{\pi}_I < \pi < \hat{\pi}_V$ the venture capitalist prefers A over I , which is



For example, “ $I \rightarrow A$ ” means that I is preferred before renegotiation, but A is chosen after renegotiation.

Fig. 5. Renegotiation outcomes.

inefficient. Renegotiation is possible, but it requires that the entrepreneur make I more attractive by reducing her stake, thus worsening her incentives. For $\hat{\pi}_A < \pi < \hat{\pi}_I$, the venture capitalist also prefers A over I . In this case, renegotiation is impossible, because the entrepreneur would have to give up so large a stake that her incentives deteriorate to the point that choosing I is no longer efficient.

Proposition 2 shows how raising the venture capitalist’s cash flow rights for A biases his exit preferences. This could lead to renegotiation, and in some cases to inefficient exit decisions. Whether this occurs in equilibrium depends on the ex ante contract, discussed next.

3.3. Optimal contracts

An important determinant of the optimal contract is the funding requirement x . Define the following critical values:

$$x_1 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A(e)} ep \, d\Omega(p, \pi) + \int_0^{\bar{p}} \int_{\hat{\pi}_A(e)}^{\bar{\pi}} u_V(e) \, d\Omega(p, \pi),$$

$$x_2 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A(e)} (p - u_E(e)) \, d\Omega(p, \pi) + \int_0^{\bar{p}} \int_{\hat{\pi}_A(e)}^{\bar{\pi}} u_V(e) \, d\Omega(p, \pi),$$

$$x_3 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A(e)} p \, d\Omega(p, \pi) + \int_0^{\bar{p}} \int_{\hat{\pi}_A(e)}^{\bar{\pi}} u_V(e) \, d\Omega(p, \pi),$$

$$x_4 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_V(e)} p \, d\Omega(p, \pi) + \int_0^{\bar{p}} \int_{\hat{\pi}_V(e)}^{\bar{\pi}} u_V(e) \, d\Omega(p, \pi),$$

where x_1 , x_2 , and x_3 are evaluated at $e = \frac{1}{2}$, and x_4 is evaluated at $e = 1$. The Appendix derives these expressions. While they might look bewildering, it is enough to know that they correspond to the critical values of the simplified model.

There is one minor difference between the simplified and the full model. In the simplified model, x_2 represents the highest x where entrepreneur control is both efficient and feasible. In the full model, x_2 only represents the highest x where entrepreneur control is efficient. Since equity stakes are not fixed to $e = e^*$ in the full model, the feasible region expands. The highest x where entrepreneur control is feasible turns out to be also x_4 ; Similarly for x_3 and contingent control.

The underlying logic of a security design problem is to first derive an optimal contract, and then show how it can be implemented using standard securities. Accordingly, Proposition 3 is divided into two parts.

Proposition 3 (Part 1: optimality). Low funding requirement ($x < x_1$):

- (i) Any optimal contract must have $\Phi(i) = e^*i$.
- (ii) A must be chosen for $\pi < \hat{\pi}_A$ and I for $\pi > \hat{\pi}_A$.

Intermediate funding requirement ($x_1 < x < x_3$):

- (i) Any optimal contract must have $\Phi(i) = e^*i$, and $F(p) > e^*p$ for some p .
- (ii) A must be chosen for $\pi < \hat{\pi}_A$ and I for $\pi > \hat{\pi}_A$.

High funding requirement ($x_3 < x < x_4$):

- (i) Any optimal contract must have $\Phi(i) = ei$, where $e > e^*$ and $F(p) = p$.
- (ii) There exists $\hat{\pi}_C > \hat{\pi}_A$, so that A is chosen for $\pi < \hat{\pi}_C$ and I for $\pi > \hat{\pi}_C$.

Note that for $x < x_3$ the outcome is always efficient. For $x_3 < x < x_4$ the outcome involves inefficient incentives (since $e > e^*$) and inefficient exit decisions (for $\hat{\pi}_A < \pi < \hat{\pi}_C$, A is chosen instead of I).

To implement the optimal contract we consider simple and intuitive securities that closely correspond to those used in practice.

Proposition 3 (Part 2: implementation). Low funding requirement ($x < x_1$):

- (i) Pure equity is optimal.
- (ii) The allocation of control rights does not matter.

Intermediate funding requirement ($x_1 < x < x_3$):

(i) Simple convertible preferred equity is optimal with $e = e^*$ and d satisfying $U_V(d, e^*) = 0$.

(ii) For $x_1 < x < x_2$, giving the entrepreneur control is optimal.

(iii) For $x_2 < x < x_3$, contingent control is optimal, so that the venture capitalist has control for $\pi < \hat{\pi}_A$ and the entrepreneur has control for $\pi > \hat{\pi}_A$.

High funding requirement ($x_3 < x < x_4$):

(i) Simple convertible preferred equity is optimal with $d = \bar{p}$ and $e > e^*$ satisfying $U_V(d, e) = 0$.

(ii) Contingent control is optimal, so that the venture capitalist has control for $\pi < \hat{\pi}_C$ and the entrepreneur has control for $\pi > \hat{\pi}_C$.

Proposition 3 contains a large number of results. Consider first the case of an intermediate funding requirement. Simple convertible preferred equity implements the optimal contract. Section 3.1 shows that, because of the two-sided incentive problem, equity is optimal in case of *I*. The question remains what cash flow rights the venture capitalist should get in case of *A*. It is better to give the venture capitalist additional cash flow rights from *A* rather than from *I*. In case of *A*, cash flow rights merely redistribute utility. In case of *I*, however, cash flow rights also affect incentives. Proposition 3 thus provides a new and simple rationale for the use of convertible preferred equity in venture capital contracts. Convertible preferred equity preserves balanced incentives if the venture remains independent, but allows the venture capitalist to extract additional rents if it is acquired. The automatic conversion in case of an IPO ensures that the additional cash flow rights are limited to acquisitions.

From Proposition 2 we also know that the use of convertible preferred equity induces biases in the exit decision. If the entrepreneur has control, these biases can be corrected through renegotiation. As in the simplified model, there exists a range $x \in [x_1, x_2]$ over which it remains feasible to implement efficient outcomes by giving the entrepreneur control. But for the range $x \in [x_2, x_3]$, implementing an efficient outcome requires contingent control.

The case of a high funding requirement is similar, except that it is harder to satisfy the venture capitalist's participation constraint. It requires increasing both the venture capitalist's cash flow rights and his control rights. In terms of cash flow rights, the optimal contract sets $e > e^*$, which creates inefficient incentives. In terms of control rights, the optimal contract increases the range over which the venture capitalist has control (i.e., $\hat{\pi}_C > \hat{\pi}_A$). This creates some inefficient exit decisions. The optimal contract trades off incentive and exit inefficiencies.

The case of a low funding requirement is very simple. Pure equity is optimal. Proposition 2 shows that the exit preferences are perfectly aligned, so that control rights do not matter. Since the venture capitalist wants to hold the efficient amount of equity ($e = e^* = \frac{1}{2}$), he needs to invest more than the minimum, namely $X = x_1 > x$.

Note that the optimal security need not be unique. Part 1 of Proposition 3 specifies the conditions than any optimal security has to satisfy. Part 2 explains how simple convertible preferred equity satisfies these conditions (for $x > x_1$). This does not exclude the possibility that other securities also satisfy those conditions. Below I explain why simple securities, such as pure debt and equity, do not satisfy the conditions of Part 1. However, it is possible to implement the optimal outcome using securities that are more complicated. These

securities differ in terms of how they structure the preferred claim in case of A . Part 2 uses simple convertible preferred equity to implement an optimal outcome. It is easy to see that other types of convertible preferred equity, such as participating convertible preferred equity, can also be used: all that matters is that the venture capitalist obtain additional cash flow rights in case of A . Since these securities are functionally equivalent, our exposition focuses only on simple convertible preferred equity. Section 4.1 provides a model extension where simple and participating convertible preferred equity are no longer perfect substitutes.

Given the richness of the model, the optimal security turns out to be remarkably simple. The optimal contract closely resembles actual venture capital contracts. In particular, the model provides a new and robust explanation for the use of convertible preferred equity. Let us briefly review why simpler securities cannot achieve the same outcome:

Why can't we just use pure equity? For $x > x_1$, the problem is that we would need to raise the venture capitalist's equity stake, which reduces the entrepreneur's incentives. Convertible preferred equity is preferable because it allocates additional cash flows to the venture capitalist only in case of A , where there are no incentive effects.

Why can't we just use pure debt? The problem is that debt doesn't allow the venture capitalist to participate in the "upside." With a double moral hazard, we want to allocate cash flows rights evenly across all possible realizations of π . Pure debt gives the venture capitalist too many cash flows rights for low π and too few for high π .

Why can't we just use a combination of pure equity and debt? The problem is that for a given value realization, the optimal contract requires different cash flow rights, depending on whether the value is realized through A or I . Formally, for some value realizations v , the optimal contract requires $\Phi(v) = ev$ but $F(v) > ev$. A simple combination of debt and equity can never create a payoff pattern that is conditional on the type of exit.

Why not make the combination of pure equity and debt contingent on the exit choice? This is essentially what convertible preferred equity does. It uses equity in case of I , but gives the venture capitalist additional cash flow rights in case of A .

4. Explaining further contract features

The main objective of the model is to explain the use of convertible securities in venture capital. I now show how the model can be extended to explain some additional contract features that are commonly used in venture capital contracts.

4.1. Different types of convertible securities

The introduction noted that there are several types of convertible securities. Proposition 3 identifies simple convertible preferred equity as an optimal security for $x > x_1$. However, this is not the only possible security. What matters for Proposition 3 is that the venture capitalist receive additional cash flow rights from A , but exactly how he receives them does not matter. The model of Section 3 is consistent with different types of convertible securities. However, it cannot explain their variety.

Empirically, there are two main types, simple convertible preferred equity (SC henceforth) and participating convertible preferred equity (PC henceforth). Kaplan and Strömberg (2003) find that 40.5% of their sample contracts use PC. Fenwick and West (2003, 2004) collect data on Silicon Valley deals, and report quarterly percentages from the

first quarter of 2002 to the second quarter of 2004. In an average quarter, 70% of their deals use PC. The lowest quarter had 56%, the highest 81%.

So far the model incorporates “late” effort after date 1. I now augment the model by also allowing for some “early” effort before date 1 (see Fig. 3) in order to be able to discriminate among the different types of convertible securities. I focus on the two main types, SC and PC. Figs. 1 and 2 show their key difference: SC concentrates the additional cash flow rights on the lower end of the value distribution, whereas PC spreads the additional cash flow rights more evenly across the value distribution. I now explain how the choice of SC versus PC is related to incentives for early effort.

Before explaining the formal model, let us briefly sketch the main intuition. Consider a contract that uses pure equity with $e = e^*$, so that we always have efficient late incentives. It turns out that such a contract provides too few early incentives for the entrepreneur: for $\theta_E > \theta_V$, the entrepreneur bears higher late effort costs, and thus obtains a lower utility from early effort. The optimal ex ante contract needs to redistribute some early incentives from the venture capitalist to the entrepreneur, without disturbing late incentives. Giving the venture capitalist additional cash flow rights on the downside reduces his early incentives, and increases the entrepreneur’s early incentives. An optimal contract accords these additional cash flow rights only in case of A , so as not to disturb late incentives in case of I . Such an optimal contract can be implemented with a convertible security. Interestingly, in the model with early incentives, the use of a convertible security is optimal for all values of x (including $x < x_1$) since there always is a need to rebalance early incentives. The main difference between SC and PC is that SC is a simpler security that can add cash flow rights only on the downside. PC offers some additional flexibility, since it can add cash flow rights both on the downside and the upside. For lower values of x , SC is sufficient, but higher values of x require the additional flexibility of PC to implement the optimal contract.

Turning now to the formal analysis, suppose that at date 0.5 (i.e., prior to date 1) the entrepreneur and venture capitalist provide private efforts $\tilde{s}_E$ and $\tilde{s}_V$, at private costs $\tilde{s}_E^2/2$ and $\tilde{s}_V^2/2$. Effort improves the odds of success, modeled as increasing the probability of a more favorable value distribution. At date 1 the values (p, π) are drawn either from a more favorable distribution $\overline{\Omega}(p, \pi)$ or from a less favorable distribution $\underline{\Omega}(p, \pi)$. The two distributions have the same support, and the ratio of densities $\overline{\omega}(p, \pi)/\underline{\omega}(p, \pi)$ is increasing in both p and π . This means that the more favorable distribution places relatively more weight on higher value realizations of p and π .⁸ The probability of a favorable distribution is given by $\tilde{s} = \tilde{\theta}_E \tilde{s}_E + \tilde{\theta}_V \tilde{s}_V$. To simplify the exposition, assume that the productivity parameters are the same before and after date 1, i.e., $\tilde{\theta}_E = \theta_E$ and $\tilde{\theta}_V = \theta_V$. The Appendix also discusses the general case of $\tilde{\theta}_E \neq \theta_E$ and $\tilde{\theta}_V \neq \theta_V$.

The Appendix provides the full analysis of the formal model, which requires some additional notation. For the main text, let us briefly sketch the structure of the equilibrium. Let $\bar{u}_E$ ($\underline{u}_E$) be the date 1 expected utility of the entrepreneur under the more (less) favorable value distribution. Same for $\bar{u}_V$ and $\underline{u}_V$. The entrepreneur chooses $\tilde{s}_E$ to

⁸The Appendix shows that the existence of an efficient SC and PC contract also requires that, for $\pi \in (0, \hat{\pi}_A)$, $\overline{\omega}(p, \pi)/\underline{\omega}(p, \pi)$ is sufficiently small for small values of p , and sufficiently large for large values of p . The intuition is that in order for SC and PC to affect early incentives, there must be a sufficient difference between the less and more favorable distribution.

maximize $U_E = \tilde{s}\bar{u}_E + (1 - \tilde{s})\underline{u}_E - \tilde{s}_E^2/2 - w$, and the venture capitalist chooses $\tilde{s}_V$ to maximize $U_V = \tilde{s}\bar{u}_V + (1 - \tilde{s})\underline{u}_V - \tilde{s}_V^2/2 - x$. The first-order conditions yield

$$\tilde{s}_E = \theta_E \alpha_E \text{ and } \tilde{s}_V = \theta_V \alpha_V, \quad \text{where } \alpha_E \equiv \bar{u}_E - \underline{u}_E \text{ and } \alpha_V \equiv \bar{u}_V - \underline{u}_V. \quad (4)$$

α_E captures the strength of the entrepreneur's early incentives. These incentives are determined by the differences in utility between the more and less favorable value distributions. The optimal contract maximizes $U_E + U_V$ subject to $U_V = 0$. For tractability reasons, the analysis is limited to the parameter region where efficient contracting is possible (i.e., where x not too large).

Remember that SC is defined by the following cash flow rights: in case of I we have $\Phi(i) = ei$; in case of A we have $F(p) = p$ for $p < d$ and $F(p) = \text{Max}[d, ep]$ for $p > d$. Define the cash flow rights of PC as follows. In case of I we have $\Phi(i) = ei$. In case of A we have $F(p) = p$ for $p < d$, $F(p) = d + e(p - d)$ for $d < p < c$ and $F(p) = ep$ if $p > c$; c is a cap on the range of acquisition prices for which the participating preferred terms apply. Such caps are frequently used in venture capital contracts.

For the next proposition let $\alpha = \alpha_E + \alpha_V = \bar{u} - \underline{u}$, which measures the total amount of available incentives.

Proposition 4 (*Type of convertible security*). (i) *The optimal contract allocates incentives according to*

$$\alpha_E = \frac{\tilde{\theta}_E^2}{\tilde{\theta}_E^2 + \tilde{\theta}_V^2} \alpha \equiv \alpha_E^* \quad \text{and} \quad \alpha_V = \frac{\tilde{\theta}_V^2}{\tilde{\theta}_E^2 + \tilde{\theta}_V^2} \alpha \equiv \alpha_V^*.$$

(ii) *There exists x_{SC} , so that for $x < x_{SC}$, the optimal contract can always be implemented with simple convertible preferred equity (SC).*

(iii) *There exists x_{PC} , with $x_{PC} > x_{SC}$, so that for $x < x_{PC}$ the optimal contract can always be implemented with participating convertible preferred equity (PC).*

The critical values x_{SC} and x_{PC} are derived in the Appendix. Proposition 4 contains several important insights. Part (i) reveals an intuitive property of the optimal distribution of incentives, which is very similar to Proposition 1. The main question is how to implement this optimal division of early incentives. We know from Proposition 1 that using pure equity with $e = e^*$ is always optimal in case of I . To maintain efficient late incentives, we need to control the early incentives through the choice of security in case of A . The proof in the Appendix explains why, for $\theta_E > \theta_V$, using pure equity at date 0 is not optimal. It would give relatively too few incentives to the entrepreneur (so that $\alpha_E < \alpha_E^*$) and relatively too many incentives to the venture capitalist (so that $\alpha_V > \alpha_V^*$). The key intuition is that while the entrepreneur receives a share $(1 - e^*)$ of the *profits* (i), she receives a relatively lower share of the *utility* (u), because for $\theta_E > \theta_V$ the entrepreneur bears a disproportionate share of effort costs.

The challenge is thus to redistribute incentives from the venture capitalist to the entrepreneur, using only cash flow rights in case of A . SC provides a natural solution, since it concentrates the venture capitalist's additional cash flow rights on low value realizations. Compensating the venture capitalist more on the downside reduces his incentives for early effort. The optimal level d_{SC} is chosen so that $\alpha_V(d_{SC}) = \alpha_V^*$. As before, there exists some critical value x_{SC} , such that the venture capitalist can break even for all $x \leq x_{SC}$.

For $x > x_{SC}$, raising d above the optimal d_{SC} would upset the balance of incentives, providing too few incentives to the venture capitalist. This is where PC becomes useful. Note that it is possible to use PC even for $x \leq x_{SC}$. Yet to the extent that PC is more complicated than SC, there is no compelling reason for using it. For $x_{SC} < x \leq x_{PC}$, however, using SC is no longer possible, whereas PC still implements the optimal contract. The key intuition is that unlike SC, PC spreads the venture capitalist's additional cash flow rights more evenly across the value distribution. It is therefore possible to increase the venture capitalist's additional cash flow rights, without affecting the distribution of incentives. The optimal PC has d_{PC} and c chosen so that $\alpha_V(d_{PC}, c) = \alpha_V^*$. Because PC increases the venture capitalist's returns without affecting optimal incentives, it expands the range over which efficient contracts can be implemented.

In Section 3, convertible preferred equity was necessary for $x > x_1$ to satisfy the venture capitalist's participation constraint. This section identifies a separate reason for the use of convertible preferred equity, which applies even for very low values of x : in addition to providing additional cash flow rights, preferred equity also helps to balance incentives between the entrepreneur and the venture capitalist.

4.2. Nonlinear and contingent contracts

Proposition 1 derives the optimality of pure equity in case of *I*. As a first approximation, this is an appropriate characterization of venture capital contracts. Naturally, a more detailed empirical investigation potentially reveals additional nonlinearities. Stock options for the entrepreneur create nonlinear payoffs. And venture capitalists sometimes hold small amounts of non-convertible debt (Kaplan and Strömberg, 2003). Interestingly, the payoff functions are typically slightly convex for the entrepreneur and slightly concave for the venture capitalist. It is also possible to create a convex payoff function for the venture capitalist, such as through warrants, but such arrangements appear to be somewhat rarer. The model can be extended to explain such nonlinearities. This section only provides an informal discussion, but a more formal derivation is available from the author upon request.

The model thus far uses the standard simplifying assumption that after date 1 there are only two possible value outcomes, IPO or failure (Casamatta, 2003; Inderst and Müller, 2004; Ueda, 2004). Consider a minimal extension, where there are three possible values: a high IPO value, a low IPO value, and failure. In a two-state model, effort affects the likelihood of an IPO, but not the size of the IPO. In a three-state model, there can be two types of private effort: one type of effort helps to reduce the likelihood of failure, which we call “survival” effort; another type of effort increases the probability of a high IPO value, which we call “value” effort. Whenever the entrepreneur's value effort is sufficiently important, it is optimal to give the venture capitalist a concave payoff and the entrepreneur a convex payoff. This requires that the optimal contract specify a bigger equity share for the entrepreneur when the IPO value is high.

Another interesting nonlinearity relates to minimum price requirements for automatic conversion. This requirement specifies that automatic conversion in case of an IPO applies only if the IPO share price is above some pre-specified minimum. Kaplan and Strömberg (2003) report that minimum price requirements are a standard feature in venture capital contracts. One puzzling aspect of minimum price requirements is that they are sometimes set at a price that is above the conversion price (i.e., the price where cash flow rights are

higher with conversion than without conversion). In this case, one might be tempted to conclude that the minimum price requirement makes the automatic conversion clause redundant. However, this conclusion turns out to be premature. The analysis of minimum price requirements is more subtle and complex. Since this has never been examined in the academic finance literature, the Appendix provides a more detailed explanation of how minimum price requirements work. The analysis provides two important insights. First, the automatic conversion clause is never redundant, even if the minimum price requirement lies above the conversion price. The reason is that automatic conversion forces investors to convert, whereas without it, investors would have an incentive to delay conversion indefinitely.⁹ Second, the main impact of a minimum price requirement is to create some additional concavity in the venture capitalist's return function.

In addition to being nonlinear, venture capital contracts make extensive use of contingent control structures. Proposition 3 shows that, for larger values of x , the optimal contract shifts control back to the entrepreneur whenever π is greater than some pre-defined threshold value. Naturally, this requires the availability of the verifiable signal (p, π) . We can justify this assumption both theoretically and empirically. Theoretically, the mechanism design literature emphasizes that if two people observe some information, it is possible under a large set of circumstances to get them to reveal this information truthfully (see Moore and Repullo, 1988). This means that for symmetrically observed information it should be possible to write contracts on this information. Empirically, we note that contingent control structures are very common in venture capital contracts. Kaplan and Strömberg (2003) show that up to 72.8% of all their sample contracts include some type of contingent contract clause. These contingency clauses typically shift control back to the entrepreneur if the venture achieves certain pre-specified milestones. The empirical use of milestones is consistent with the current model, which predicts that the entrepreneur regains control for sufficiently good realizations of π .

The model also makes a prediction about the optimal structure of such milestones. The milestone pertains to the expected value of the firm as an independent entity (π). The Appendix shows that the optimal threshold value $\hat{\pi}_A$ (or $\hat{\pi}_C$) is increasing in p . We can think of p as measuring the asset value of the firm. Beyond asset values, π includes additional information about the ability of the entrepreneur to manage the venture up to an IPO. Optimal milestones should pick up not only asset values but also information about the viability of the firm as an independent entity. Consistent with this, Kaplan and Strömberg report the use of milestones that concern the ability of the entrepreneur to manage the venture, such as whether a founder can assemble a management team.

Another interesting empirical finding is that cash flow rights can be contingent on milestones. Gompers (1997), for example, finds that 35% of his sample contracts have early conversion clauses. This means that conversion to common stock does not have to await the IPO, but is automatic upon achievement of the milestone. Proposition 3 focuses on contingent control. It is easy to see that the same outcome can also be achieved with contingent cash flow rights. Suppose the venture capitalist has control. Consider a convertible preferred equity with an early conversion clause, that automatically converts for $\pi > \hat{\pi}_A$. Proposition 2 implies that for $\pi > \hat{\pi}_A$ the venture capitalist chooses I without

⁹Delaying conversion can be inefficient. Stock market investors might prefer a more transparent capital structure. In addition, Black and Gilson (1997) argue that it is efficient that the entrepreneur regain control rights at the time of the IPO, which requires conversion of preferred stock.

any need for renegotiation. And for $\pi < \hat{\pi}_A$ the venture capitalist chooses A , continuing to get his additional cash flow rights.¹⁰

4.3. Further empirical predictions

In addition to the numerous connections already mentioned between the model and the empirical facts, the model generates additional testable implications about venture capital contracts. Two central predictions of the model relate to the terms of convertible preferred equity. First, the model predicts that the amount of equity after conversion (e in the model) depends on the relative abilities of the entrepreneur and venture capitalist. Consistent with this, Kaplan and Strömberg (2004) find that founders get less equity when they have greater internal risks (such as incomplete or inexperienced management teams). And Hsu (2004) shows that companies retain less equity when dealing with higher-reputation venture capitalists.

Second, the model predicts that the strength of the downside protection (d in the model) is increasing in the external financing requirement (x), or decreasing in the expected returns of the venture (higher k or lower p 's and π 's). Kaplan and Strömberg (2004) find that greater internal and external risks are associated with stronger downside protection. The cyclical patterns of liquidation preferences is also consistent with this result. Multiple liquidation preferences were rare in the boom period 1997–2000, increased dramatically in the bust years of 2001–2002, and recently came down again (see Fenwick and West, 2003, 2004). Along similar lines, the theory predicts that entrepreneurs are more likely to retain control rights when they have fewer risks (or require less external financing), a pattern also confirmed by Kaplan and Strömberg.

The paper also generates some hypotheses that have not yet been tested. Besides the predictions about optimal milestones, another interesting hypothesis concerns the relative frequency of exits. In particular, the model predicts that the stronger the venture capitalist's control rights, the more likely it is that exit will occur by acquisition rather than IPO.

5. Conclusion

This paper develops a model that jointly explains two central aspects of venture capital financing: how venture capitalists make exit decisions, and why they use contracts with convertible preferred equity and contingent control rights. A key insight is that convertible preferred equity allocates different cash flow rights for acquisitions and IPOs. The analysis also explains some puzzling contract features, such as automatic conversion in case of IPO or contingent control structures.

In the model, contracts govern future ownership decisions. This is an interesting departure from the standard assumption that ownership decisions are made *ex ante*, and opens up a broader set of research questions: What are the dynamics of ownership decisions? What generates disagreement about the optimal timing of exit? And what contractual features can mitigate these problems?

¹⁰Strictly speaking, this argument applies only to $x \in [x_2, x_3]$. The working paper version discusses a minor complication of using contingent cash flow rights for $x \in [x_3, x_4]$.

Appendix

Appendix A.1. Proof of Proposition 1

Throughout the appendix I refer to the entrepreneur as E and the venture capitalist as V . Consider the private effort choices after date 1. Let $e(i) = \Phi(i)/i$ denote V 's fraction of returns in I . Below I show that the optimal e is always a constant. Given $\pi = \sigma i$, s_E and s_V are respectively chosen to maximize $u_E(e) = (\theta_E s_E + \theta_V s_V)(1 - e)\pi - s_E^2/2$ and $u_V(e) = (\theta_E s_E + \theta_V s_V)e\pi - s_V^2/2$. The first-order conditions yield $s_E = \theta_E(1 - e)\pi$ and $s_V = \theta_V e\pi$. This implies $u_E(e) = \pi^2[(\theta_E^2(1 - e) + \theta_V^2 e)(1 - e) - \theta_E^2/2(1 - e)^2]$ and $u_V(e) = \pi^2[(\theta_E^2(1 - e) + \theta_V^2 e)e - (\theta_V^2/2)e^2]$. To find the efficient value of e , I maximize $u(e) \equiv u_E(e) + u_V(e) = \pi^2[(\theta_E^2(1 - e) + \theta_V^2 e) - \theta_E^2/2(1 - e)^2 - (\theta_V^2/2)e^2]$ w.r.t. e . The first-order condition is given by $\pi^2[\theta_V^2 - \theta_E^2 - \theta_E^2(1 - e) - \theta_V^2 e] = 0$, which yields after transformation $e^* = \theta_V^2/(\theta_E^2 + \theta_V^2)$. For any $e < e^*$, V can increase his stake to e^* , thereby increasing efficiency. But for $e > e^*$, the joint utility u is decreasing in e .

We immediately note that e^* does not depend on π . This means that an efficient contract requires $\Phi(i) = e^*i$, where e^* is a constant. Thus equity is the optimal security. The same argument continues to apply even if it necessary to give V additional cash flow rights in case of I (such as for $x_3 > x$ in Proposition 3). To see that it is always optimal to keep e constant across different values of π , consider any distribution of π . We want to maximize $\int u_E(e(\pi), \pi)$, subject to $\int u_V(e(\pi), \pi)$ greater than or equal to some constant. Pointwise maximizing $e(\pi)$ yields for every π a first-order condition $\omega(\pi)u'_E + \lambda\omega(\pi)u'_V = 0$. Using $u'_E = \pi^2[-\theta_E^2(1 - e) + \theta_V^2(1 - 2e)]$ and $u'_V = \pi^2[\theta_E^2(1 - 2e) + \theta_V^2 e]$ we get after transformations $\bar{e} = [\theta_V^2 + (\lambda - 1)\theta_E^2]/[(2 - \lambda)\theta_V^2 + (2\lambda - 1)\theta_E^2]$ for every π . Again, this is independent of π , so that the optimal security remains pure equity.

For all proofs I focus on the more relevant case of $\theta_E \geq \theta_V$, so that E 's effort is relatively more important than V 's. The case of $\theta_E < \theta_V$ is analogous, and I leave its derivation to the interested reader. For the asymmetric case of $\theta_E > \theta_V$ we can find the maximal level of e from setting $u'_V = \pi^2[\theta_E^2(1 - 2e) + \theta_V^2 e] = 0$. This yields $\bar{e} = \theta_E^2/(2\theta_E^2 - \theta_V^2)$. For $e > \bar{e}$, V would prefer to relinquish equity, in order to provide better incentives to E . In other words, $e > \bar{e}$ represents a backward-bending portion of the utility frontier. For the symmetric case of $\theta_E = \theta_V$, $e^* = \frac{1}{2}$ and $\bar{e} = 1$.

For future reference, I show that for $\theta_E > \theta_V$ we have $u_E(e^*) < (1 - e^*)u(e^*)$ and $u_V(e^*) > e^*u(e^*)$. To see this, note that $u_E(e^*) = (1 - e^*)s\pi - s_E^2/2 < (1 - e^*)u(e^*) = (1 - e^*)[s\pi - s_E^2/2 - s_V^2/2] \Leftrightarrow -e^*s_E^2 < -s_V^2(1 - e^*)$. Using $s_E = \theta_E(1 - e^*)\pi$ and $s_V = \theta_V e^*\pi$ this is equivalent to $-\theta_E^2(e^*)(1 - e^*)^2 < -\theta_V^2(e^*)^2(1 - e^*) \Leftrightarrow \theta_E > \theta_V$. The intuition is that while E and V share profits in proportions $(1 - e^*)$ and e^* , they do not share effort costs in those same proportions. For $\theta_E > \theta_V$, E bears a disproportionate share of the effort cost. This means that in utility terms, E receives a share that is less than $(1 - e^*)$ of the total utility $u(e^*)$. I will use this result in Propositions 2 and 3.

The model uses a linear quadratic specification. [Inderst and Müller \(2004\)](#) and [Repullo and Suarez \(2004\)](#) also consider a Cobb-Douglass model, where there are complementarities between private effort. I briefly show how the model is also robust to such an alternative specification. Suppose that s is given by $s = \theta s_V s_E^{1-\gamma}$, where γ is a measure of the relative contribution of V . Using $u_V = s e \pi - s_V^2/2$ and $u_E = s \bar{e} \pi - s_E^2/2$, I obtain as first-order

conditions $\gamma s_V^{\gamma-1} s_E^{1-\gamma} \theta e \pi = s_V$ and $(1-\gamma) s_V^\gamma s_E^{-\gamma} \theta \bar{e} \pi = s_E$. After some tedious transformations I obtain $s_V = \theta \pi (\bar{\gamma} \bar{e})^{(1-\gamma)/2} (\gamma e)^{(1+\gamma)/2}$ and $s_E = \theta \pi (\bar{\gamma} \bar{e})^{(2-\gamma)/2} (\gamma e)^{\gamma/2}$, and therefore $s = \theta^2 \pi (\bar{\gamma} \bar{e})^{1-\gamma} (\gamma e)^\gamma$, $u_V = \theta^2 \pi^2 (\bar{\gamma} \bar{e})^{1-\gamma} (\gamma e)^\gamma \bar{\gamma} e$ and $u_E = \theta^2 \pi^2 (\bar{\gamma} \bar{e})^{1-\gamma} (\gamma e)^\gamma \gamma \bar{e}$. To find the optimal e , I maximize $u_E + u_V = \theta^2 \pi^2 (\bar{\gamma} \bar{e})^{1-\gamma} (\gamma e)^\gamma [e + \gamma - 2\gamma e]$. Further calculations reveal that the optimal e is given by $e^* = [1 - 2\gamma^2 - 2\gamma + \sqrt{1 - 4\gamma + 8\gamma^2 - 8\gamma^3 + 4\gamma^4}]/(4 - 8\gamma)$. The optimal e^* is again independent of π . It follows that equity remains the optimal choice of security.

Appendix A.2. Proof of Proposition 2

Let us use the notation $u_E = \pi^2 z_E$, $u_V = \pi^2 z_V$ and $u = \pi^2 z$. The advantage of this notation is that the z functions do not depend on π . It is also useful to define $\Pi = \pi^2$, and to express all critical values of π as critical values of Π . Only for the proof of this proposition, it is also useful to rewrite cash flow rights in case of A as $F(p) = (e^* + \varepsilon)p$.

The critical value $\hat{\Pi}_E$ (corresponding to $\hat{\pi}_E = \sqrt{\hat{\Pi}_E}$) is found from $u_E(e^*) = p - F(p) \Leftrightarrow \Pi_E z_E(e^*) = (1 - e^* - \varepsilon)p \Leftrightarrow \hat{\Pi}_E = (1 - e^* - \varepsilon)p/z_E(e^*)$. The critical value $\hat{\Pi}_V$ is found from $u_V(e^*) = F(p) \Leftrightarrow \hat{\Pi}_V = (e^* + \varepsilon)p/z_V(e^*)$. To find the critical value $\hat{\Pi}_A$, note that renegotiation to A is possible whenever $p \geq u(e^*)$, i.e., when the joint value under A exceeds the joint value under I . The critical value $\hat{\Pi}_A$ is thus given by $\hat{\Pi}_A = p/z(e^*)$. Finally, renegotiation to I requires that $u(e^R) \geq p$ where e^R is the division of equity that obtains after renegotiation. If there is no binding wealth constraint, then $e^R = e^*$, but if E is wealth constrained, then $e^R > e^*$. In this case the critical value $\hat{\Pi}_I$ is implicitly given by $u(e^R) = p$ where $u_E(e^R) = (1 - e^* - \varepsilon)p$.

Let us focus first on the symmetric case of $\theta_E = \theta_V$. Consider the case of straight equity, where $e = e^* = \frac{1}{2}$ and $F(p) = p/2 \Leftrightarrow \varepsilon = 0$. Note that we have perfect symmetry, so that E and V both get $u(e^*)/2$ in case of I and $p/2$ in case of A . E and V both prefer I over A whenever $u(e^*) > p \Leftrightarrow \Pi > \hat{\Pi}_A$. It follows that $\hat{\Pi}_E = \hat{\Pi}_A = \hat{\Pi}_I = \hat{\Pi}_V$. This establishes part (i).

Consider next the case of $F(p) > e^*p \Leftrightarrow \varepsilon > 0$. Note that $\hat{\Pi}_V = (1 + 2\varepsilon)(p/z(e^*)) > p/z(e^*) = \hat{\Pi}_A$ and $\hat{\Pi}_E = (1 - 2\varepsilon)(p/z(e^*)) < p/z(e^*) = \hat{\Pi}_A$. To show that $\hat{\Pi}_A < \hat{\Pi}_I$, note that $\hat{\Pi}_I$ satisfies $u(e^R) = p \Leftrightarrow \hat{\Pi}_I = \frac{p}{z(e^R)}$. For a binding wealth constraint, $e^R > e^* \Leftrightarrow z(e^R) < z(e^*) \Leftrightarrow \hat{\Pi}_I = p/z(e^R) > p/z(e^*) = \hat{\Pi}_A$. To show that $\hat{\Pi}_I < \hat{\Pi}_V$, note that $\hat{\Pi}_I$ satisfies $u_V(e^R) = F(p) \Leftrightarrow \hat{\Pi}_I = F(p)/z_V(e^R)$. For $e^R > e^*$, $z_V(e^R) > z_V(e^*) \Leftrightarrow \hat{\Pi}_I = F(p)/z_V(e^R) < F(p)/z_V(e^*) = \hat{\Pi}_V$. This establishes part (ii).

To derive parts (iii) and (iv) it is useful to define five parameter regions that correspond to Fig. 5: Region (1) corresponds to $\Pi < \hat{\Pi}_E$, region (2) corresponds to $\hat{\Pi}_E < \Pi < \hat{\Pi}_A$, region (3) corresponds to $\hat{\Pi}_A < \Pi < \hat{\Pi}_I$, region (4) corresponds to $\hat{\Pi}_I < \Pi < \hat{\Pi}_V$, and region (5) corresponds to $\Pi > \hat{\Pi}_V$.

Let us begin with E control. For region (1), E prefers A which is also efficient. Denote this by $A \rightarrow A$. For region (2), $\Delta_E > 0$, so that prior to renegotiation, E prefers I . Also, $\Delta(e^*) < 0$ so that renegotiation to A is feasible, and will therefore occur. The renegotiation outcome is thus given by $I \rightarrow A$. For regions (3), (4), and (5), $\Delta_E > 0$, so that prior to renegotiation, E prefers I , which is efficient and implies $I \rightarrow I$.

If V has control, consider regions $\langle 1 \rangle$ and $\langle 2 \rangle$. V prefers A , which is efficient, so that $A \rightarrow A$. In region $\langle 3 \rangle$, V prefers A even though it is inefficient. Renegotiation to I is not feasible, so that $A \rightarrow A$ obtains. The intuition why renegotiation fails is that it would require such a high value of e^R , so that $u(e^R) < p < u(e^*)$. I looks efficient before renegotiation, but would actually be inefficient under the terms of renegotiation. In region $\langle 4 \rangle$, V again prefers A before renegotiation. Renegotiation to I is feasible, so that $A \rightarrow I$. Since $e^R > e$, renegotiation creates inefficient incentives. Unlike in region $\langle 3 \rangle$ we have $p < u(e^R) < u(e^*)$, so that I remains more efficient, even under the terms of renegotiation. In region $\langle 5 \rangle$, V prefers I before renegotiation, which is efficient, so that $I \rightarrow I$.

To complete the proof let us examine the asymmetric model with $\theta_E > \theta_V$. The only difference is that from Proposition 1, $u_E(e^*) < (1 - e^*)u(e^*)$ and $u_V(e^*) > e^*u(e^*)$. Define $\eta = z_V(e^*)/e^*z(e^*) > 1$ so that $z_E(e^*) = (1 - \eta e^*)z(e^*)$ and $z_V(e^*) = \eta e^*z(e^*)$. Thus $\hat{\Pi}_V = (e^* + \varepsilon)p/\eta e^*z(e^*)$ and $\hat{\Pi}_E = (1 - e^* - \varepsilon)p/(1 - \eta e^*)z(e^*)$. It follows that $\hat{\Pi}_V > \hat{\Pi}_A = p/z(e^*) \Leftrightarrow \varepsilon > (\eta - 1)e^*$ and $\hat{\Pi}_E < \hat{\Pi}_A \Leftrightarrow (1 - e^* - \varepsilon)/(1 - \eta e^*) < 1 \Leftrightarrow \varepsilon > (\eta - 1)e^*$. For higher values of ε (namely $\varepsilon > (\eta - 1)e^*$), $\hat{\Pi}_E < \hat{\Pi}_A < \hat{\Pi}_V$. The proof is then analogous to the symmetric case (note that the proof for $\hat{\Pi}_A < \hat{\Pi}_I < \hat{\Pi}_V$ never uses $\theta_E = \theta_V$). For low values of ε , $\hat{\Pi}_V < \hat{\Pi}_A < \hat{\Pi}_E$. This never creates any inefficiencies in the renegotiation game. For $\hat{\Pi}_V < \Pi < \hat{\Pi}_A < \hat{\Pi}_E$, E prefers A over I which is efficient. V prefers I over A which is inefficient. However, renegotiating to the efficient A is always possible: E simply gives up some cash flow rights for A , so that $p - F^R = u_E(e^*)$ and $F^R > u_V(e^*)$. For $\hat{\Pi}_V < \hat{\Pi}_A < \Pi < \hat{\Pi}_E$, V prefers I over A which is efficient. E prefers A over I which is inefficient. However, renegotiating to the efficient I is always possible: V keeps his optimal equity stake e^* and simply makes a transfer payment ς to E , i.e. $u_E(e^*) + \varsigma = p - F$ and $u_V(e^*) - \varsigma > F$. (Note that this last part ignores $\hat{\Pi}_I$ because for $\varepsilon < (\eta - 1)e^*$, $e^R = e^*$, so that $\hat{\Pi}_I = \hat{\Pi}_A$.)

Appendix A.3. Proof of Proposition 3

The maximization problem consists of choosing optimal cash flow rights (F, Φ) and optimal control rights. Denote the control structure by $\Psi = 0$ if E has control and $\Psi = 1$ if V has control. Ψ could depend on the realization of (p, π) .

To simplify the notation I omit all the integration terms $d\Omega(p, \pi)$. With the regions $\langle 1 \rangle$, $\langle 2 \rangle$, $\langle 3 \rangle$, $\langle 4 \rangle$, and $\langle 5 \rangle$ from the proof of Proposition 2, the ex ante utilities of $j = E, V$ are given by

$$U_j = \int_0^{\bar{p}} \int_0^{\hat{\pi}_E} U_j^{(1)} + \int_0^{\bar{p}} \int_{\hat{\pi}_E}^{\hat{\pi}_A} U_j^{(2)} + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\hat{\pi}_I} U_j^{(3)} + \int_0^{\bar{p}} \int_{\hat{\pi}_I}^{\hat{\pi}_V} U_j^{(4)} + \int_0^{\bar{p}} \int_{\hat{\pi}_V}^{\bar{\pi}} U_j^{(5)},$$

where

$$U_E^{(1)} = p - F, \quad U_V^{(1)} = F,$$

$$U_E^{(2)} = \Psi(p - F) + (1 - \Psi)u_E(e), \quad U_V^{(2)} = \Psi F + (1 - \Psi)(p - u_E(e)),$$

$$U_E^{(3)} = \Psi(p - F) + (1 - \Psi)u_E(e), \quad U_V^{(3)} = \Psi F + (1 - \Psi)u_V(e),$$

$$U_E^{(4)} = \Psi(p - F) + (1 - \Psi)u_E(e), \quad U_V^{(4)} = \Psi u_V(e^R) + (1 - \Psi)u_V(e),$$

$$U_E^{(5)} = u_E(e), \quad U_V^{(5)} = u_V(e).$$

From Proposition 1, any optimal contract uses a constant e for all values of π . The efficient value is $e^* = \frac{1}{2}$. From Proposition 2, any optimal contract would want to have A for $\pi < \hat{\pi}_A$ and I for $\pi > \hat{\pi}_A$ (where $\hat{\pi}_A$ is evaluated at $e = \frac{1}{2}$). This is possible for all $x < x_3$. To obtain x_3 , use the highest possible returns for V within these constraints, given by $F = p$ and $e = \frac{1}{2}$, i.e., $x_3 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} p + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V$. To obtain x_1 , use pure equity with $F = \frac{p}{2}$ and $e = \frac{1}{2}$, implying $x_1 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} p/2 + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V$. To obtain x_4 , identify the most favorable contract for V , given by $F = p$ and $e = 1$. Evaluating this contract at $\hat{\pi}_I = \hat{\pi}_V = p/z_V$, we get $x_4 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_V} p + \int_0^{\bar{p}} \int_{\hat{\pi}_V}^{\bar{\pi}} u_V$.

For $x < x_1$ we can always use $F = p/2$ and $e = \frac{1}{2}$ and satisfy V 's participation constraint by requiring an investment $X = x_1$, guaranteeing $U_V = 0$. For $x_1 < x < x_3$ it is necessary to set $F > p/2$ for some values of p , to satisfy V 's participation constraint. And for $x_3 < x < x_4$, it is always optimal to set $F = p$.

To complete the optimality part of the Proposition requires only explaining the optimal contract for $x_3 < x < x_4$. We need to find the optimal choice of e , as well as the optimal exit decision. To represent the optimal exit decision, I claim that it can always be described by some $\hat{\pi}_C$ with $\hat{\pi}_A < \hat{\pi}_C \leq \hat{\pi}_I$, so that A is chosen for $\pi < \hat{\pi}_C$ and I is chosen for $\pi > \hat{\pi}_C$. The ex ante optimization problem is to maximize $L = U_E + \lambda(U_V - x)$ where $U_E = \int_0^{\bar{p}} \int_{\hat{\pi}_C(p)}^{\bar{\pi}} u_E(e)$ and $U_V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_C(p)} p + \int_0^{\bar{p}} \int_{\hat{\pi}_C(p)}^{\bar{\pi}} u_V(e)$. For this maximization, we can choose e and $\hat{\pi}_C$, both of which can depend on the verifiable state (p, π) . The first-order condition for $e(p, \pi)$ is given by $dL/de = \omega(p, \pi)u'_E(e) + \lambda\omega(p, \pi)u'_V(e) = 0$. It is useful to write $\lambda = 1 + \mu(e)$, where $\mu(e) \equiv -u'_E/u'_V - 1$. Note that the optimal e does not depend on the particular realization of π (see also Proposition 1). Note also that $\lambda > 1 \Leftrightarrow \mu > 0 \Leftrightarrow e > \frac{1}{2}$. The first-order condition for $\hat{\pi}_C(p)$ is given by $dL/d\hat{\pi}_C = -\omega(p, \hat{\pi}_C)u_E(e; \hat{\pi}_C) + \lambda\omega(p, \hat{\pi}_C)[p - u_V(e; \hat{\pi}_C)]$. Rewriting this as $\lambda[p - \hat{\pi}_C z_V(e)] = \hat{\pi}_C z_E(e)$ and using $\lambda = 1 + \mu(e)$ implies $\hat{\pi}_C = (p + \mu p)/(z(e) + \mu z_V(e))$. Thus, $\hat{\pi}_C = \hat{\pi}_A$ for $\mu(e) = 0$ and $\hat{\pi}_C > \hat{\pi}_A$ for $\mu(e) > 0$. And $d\hat{\pi}_C/dp = (1 + \mu)/(z(e) + \mu z_V(e)) > 0$, which shows that $\hat{\pi}_C$ is an increasing function of p .

To see that the optimal exit structure satisfies $\hat{\pi}_C \leq \hat{\pi}_I$, suppose $\hat{\pi}_C > \hat{\pi}_I$. For $F = p$, $e^R = 1 > e$. The ex ante utilities are given by $U_E = \int_0^{\bar{p}} \int_{\hat{\pi}_C(p)}^{\bar{\pi}} u_E(e)$ and $U_V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_I} p + \int_0^{\bar{p}} \int_{\hat{\pi}_I}^{\hat{\pi}_C(p)} u_V(e^R) + \int_0^{\bar{p}} \int_{\hat{\pi}_C(p)}^{\bar{\pi}} u_V(e)$. This is inefficient, since e is not a constant. It would be more efficient to set $\hat{\pi}_C = \hat{\pi}_I$ and choose a constant e' with $e < e' < \bar{e}$.

To see that the optimal exit structure divides exits into exactly two areas of π , suppose there were more than two areas. Clearly, for any $\pi < \hat{\pi}_A$, both E and V always prefer A , so this must be efficient. For any $\pi > \hat{\pi}_I$, both prefer I , so this must be efficient too. We can thus focus on $\hat{\pi}_A < \pi < \hat{\pi}_I$, where the two may disagree on having A or I . With more than two areas, there always exists $\pi' < \pi''$ so that I is chosen for π' and A (with $F = p$) for π'' . It would be better to switch the outcomes for these two values of π . This would give V the same utility of p , but give E a higher utility, since $u_E(e, \pi'') > u_E(e, \pi')$. It follows that having more than two areas is never efficient. Thus, $\hat{\pi}_C$ is the unique critical value.

Having described the properties of the optimal contract, let us now look for securities that naturally implement this optimal contract. As in any security design problem, there can be multiple ways of implementing the same optimal contract. We always look for the simplest and most intuitive securities (e.g., we don't use contingent contracts when it is possible to use a non-contingent contract). For $x < x_1$ it is optimal to use pure equity with $F = p/2$ and $e = \frac{1}{2}$. From Proposition 2, pure equity perfectly aligns E 's and V 's preferences between A and I . Irrespective of who has control, A is chosen for $\pi < \hat{\pi}_A$ and I is chosen for $\pi > \hat{\pi}_A$. To satisfy the constraint $U_V = X$, set $X = x_1 > x$.

Consider now E control with $x > x_1$. Using $e = \frac{1}{2}$ clearly is optimal. We can then set d so that $U_V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_E} \text{Max}[\text{Min}[p, d], p/2] + \int_0^{\bar{p}} \int_{\hat{\pi}_E}^{\hat{\pi}_A} (p - u_E) + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V - x = 0$ (evaluated at $e = \frac{1}{2}$). This generates an efficient outcome, and is feasible for $x \leq x_2$, where $d = \bar{p}$ and $\hat{\pi}_E = 0$, so that $x_2 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} (p - u_E) + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V$.

For $x_2 < x < x_3$, V needs additional cash flow rights. Giving V full control is never efficient. From Proposition 2, raising F above $p/2$ always creates some inefficient exits. And from Proposition 1, raising e above $e^* = \frac{1}{2}$ always creates inefficient incentives. To implement the efficient outcome, it becomes necessary to consider contingent control. We can use simple convertible preferred equity and set d so that $U_V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} \text{Max}[\text{Min}[p, d], p/2] + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V - x = 0$ (evaluated at $e = \frac{1}{2}$). And we need contingent control rights where V has control for $\pi < \hat{\pi}_A$ and E for $\pi > \hat{\pi}_A$. From Proposition 2, for $\pi < \hat{\pi}_A$, V always chooses the efficient A . And for $\pi > \hat{\pi}_A$, E always chooses the efficient I .

For $x_3 < x < x_4$, we can implement the optimal contract in a similar way, where contingent control rights depend on the threshold $\hat{\pi}_C$. The optimal security requires $F = p$, so that $d = \bar{p}$ (this is equivalent to V taking full ownership in case of A). The optimal value of e satisfies $U_V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_C} p + \int_0^{\bar{p}} \int_{\hat{\pi}_C}^{\bar{\pi}} u_V(e) - x = 0$.

The main text also discusses contingent cash flow rights. If $x \in [x_2, x_3]$, then V can have control, along with an early conversion clause, i.e., $F(p) > p/2$ for some p if $\pi < \hat{\pi}_A$, but $F(p) = p/2$ for all p if $\pi > \hat{\pi}_A$. It is immediate that this implements the optimal contract. For $x \in [x_3, x_4]$ there may be a minor complication with a simple early conversion clause. Setting $F(p) = ep$ for all p if $\pi > \hat{\pi}_C$ might not be enough to guarantee that V chooses I without renegotiation, because for $e > \frac{1}{2}$, the possibility that $ep > u_V(e)$ at $\hat{\pi}_C$ cannot be excluded. If this were the case, it would obviously still be possible to create a slightly more advanced contingent cash flow structure, where $F(p)$ converts to something lower than ep , so as to ensure that V prefers I before renegotiation.

The model assumes that V has all the bargaining power. The only case where renegotiation occurs in equilibrium is for $x \in [x_1, x_2]$. If E has some bargaining power, she extracts more rents from renegotiation. This shrinks the region where E control is feasible.

It is easy to see that x_2 is given by $x_2 = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} u_V + \beta(p - u) + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V$, where $\beta \in [0, 1]$ measures the bargaining power of V .

Finally, the proof focuses on the symmetric case of $\theta_E = \theta_V$. Albeit tedious, it is straightforward to verify that all the results of Proposition 3 continue to hold for the asymmetric case $\theta_E > \theta_V$, obviously replacing all $e = \frac{1}{2}$ with $e = e^*$. One minor difference concerns the analysis with $x < x_1$. The proof of Proposition 2 shows that with pure equity contracts, $\hat{\pi}_V < \hat{\pi}_A < \hat{\pi}_E$, and all renegotiation is efficient. However, renegotiation affects

the distribution of utility, implying that x_1 now depends on whether E and V have control. Straightforward calculations show that when E has control, $x_1^E = \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} p/2 + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} p/2 + \int_0^{\bar{p}} \int_{\hat{\pi}_E}^{\bar{\pi}} u_V$, and when V has control, $x_1^V = \int_0^{\bar{p}} \int_0^{\hat{\pi}_V} p/2 + \int_0^{\bar{p}} \int_{\hat{\pi}_V}^{\hat{\pi}_A} [p - u_E] + \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V$. It is easy to verify that $x_1^E < x_1^V < x_2$. This does not affect any other parts of Proposition 3.

Appendix A.4. Proof of Proposition 4

Let us first introduce some useful notation:

$$\begin{aligned}\bar{u}_E^A &= \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} (p - F) d\bar{\Omega}(p, \pi), & \bar{u}_V^A &= \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} F d\bar{\Omega}(p, \pi), \\ \bar{u}_E^I &= \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_E(e^*) d\bar{\Omega}(p, \pi), & \bar{u}_V^I &= \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V(e^*) d\bar{\Omega}(p, \pi), \\ \underline{u}_E^A &= \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} (p - F) d\underline{\Omega}(p, \pi), & \underline{u}_V^A &= \int_0^{\bar{p}} \int_0^{\hat{\pi}_A} F d\underline{\Omega}(p, \pi), \\ \underline{u}_E^I &= \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_E(e^*) d\underline{\Omega}(p, \pi), & \underline{u}_V^I &= \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V(e^*) d\underline{\Omega}(p, \pi).\end{aligned}$$

Also let $\bar{u}_E = \bar{u}_E^A + \bar{u}_E^I$, $\bar{u}_V = \bar{u}_V^A + \bar{u}_V^I$, and $\bar{u} = \bar{u}_E + \bar{u}_V$; similarly for $\underline{u}_E$, $\underline{u}_V$, and $\underline{u}$. Let $\tilde{u}_E^A = \tilde{s}\bar{u}_E^A + (1 - \tilde{s})\underline{u}_E^A$, where $\tilde{s}$ is evaluated at the optimal values $\tilde{s}_E$ and $\tilde{s}_V$; similar for I and for V . In addition, define $\hat{\Omega}(p, \pi) = \bar{\Omega}(p, \pi) - \underline{\Omega}(p, \pi)$ and $\hat{\omega}(p, \pi) = \bar{\omega}(p, \pi) - \underline{\omega}(p, \pi)$.

Central to the analysis of early incentives is the difference between the more and less favorable value distributions. For this, define $\alpha_E^A = \bar{u}_E^A - \underline{u}_E^A$ (same for I and for V), $\alpha^A = \alpha_E^A + \alpha_V^A = \bar{u}^A - \underline{u}^A$ (same for I), $\alpha_E = \alpha_E^A + \alpha_E^I = \bar{u}_E - \underline{u}_E$ (same for V), and $\alpha = \alpha_E + \alpha_V = \bar{u} - \underline{u}$.

Part (i) involves examining the properties of the optimal contract that maximizes $L = \tilde{s}^*\bar{u} + (1 - \tilde{s}^*)\underline{u} - (\tilde{s}_E^*)^2/2 - (\tilde{s}_V^*)^2/2$. Using $\tilde{s}_E^* = \theta_E\alpha_E$ and $\tilde{s}_V^* = \theta_V\alpha_V$, we can rewrite this as $L = \underline{u} + (\theta_E^2\alpha_E + \theta_V^2\alpha_V)\alpha - (\theta_E\alpha_E)^2/2 - (\theta_V\alpha_V)^2/2$. Consider an increase dF . Using $d\alpha_E/dF = -d\alpha_V/dF$ we get $dL/dF = (\theta_E^2(d\alpha_E/dF) - \theta_V^2(d\alpha_E/dF)\alpha - \theta_E^2\alpha_E(d\alpha_E/dF) + \theta_V^2\alpha_V(d\alpha_E/dF)) = 0$. Using $\alpha_E = \alpha - \alpha_V$, we get after simple transformations $\alpha_V = \alpha\theta_V^2/(\theta_V^2 + \theta_E^2) = e^*\alpha$ and $\alpha_E = \alpha\theta_E^2/(\theta_V^2 + \theta_E^2) = (1 - e^*)\alpha$.

For parts (ii) and (iii) it is useful to first examine the model with pure equity (using $e = e^*$). As in the proof of Proposition 2, we can write $u_E(e) = \pi^2 z_E(e)$ and $u_V(e) = \pi^2 z_V(e)$, where z_E and z_V are independent of π . From the proof of Proposition 1, $\theta_E > \theta_V \Leftrightarrow u_V(e^*) > e^*u(e^*) \Leftrightarrow z_V(e^*) > e^*z(e^*)$. We then have $\alpha_V^I = \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} u_V(e^*) d\hat{\Omega} = z_V(e^*) \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} \pi^2 d\hat{\Omega} > e^*z(e^*) \int_0^{\bar{p}} \int_{\hat{\pi}_A}^{\bar{\pi}} \pi^2 d\hat{\Omega} = e^*\alpha^I$. Moreover, with pure equity we have $\alpha_V^A = e^*\alpha^A$. Thus $\alpha_V = \alpha_V^A + \alpha_V^I > e^*\alpha^A + e^*\alpha^I = e^*\alpha$. This establishes that pure equity gives V too many incentives, i.e., α_V is too large. And it gives E too few incentives, i.e., α_E is too small. The intuition is again that while E and V share profits (π) in proportions $1 - e^*$

and e^* , E bears a relatively larger share of the effort costs, giving her less than $1 - e^*$ of a share in the total utility (u) in case of I .

In order to correct ex ante incentives we do not want to change $e = e^*$, since this would cause ex post inefficiencies. We therefore correct incentives by adjusting the returns in case of A . Specifically, since $\alpha_V^I > e^* \alpha^I$, we want a lower value of α_V^A . This can be achieved by increasing $F(p)$ for low values of p . Specifically, consider the effect on α_V^A of increasing $F(p)$ for some p , i.e., $d\alpha_V^A/dF = d\bar{u}_V^A/dF - d\underline{u}_V^A/dF = \int_0^{\pi_A} \hat{\omega}(p, \pi) d\pi$. Since $\bar{\omega}(p, \pi)/\underline{\omega}(p, \pi)$ is increasing in p , $\hat{\omega}(p, \pi) = \bar{\omega}(p, \pi) - \underline{\omega}(p, \pi)$ is also increasing in p , ranging from negative values for low p to positive values for high p . Using the mean value theorem, there exists p' so that $\int_0^{\pi_A} \hat{\omega}(p', \pi) d\pi = 0$. For all $p < p'$, $d\alpha_V^A/dF < 0$; and for all $p > p'$, $d\alpha_V^A/dF > 0$.

To implement an efficient contract with SP, note that for any $d < p'$, increasing d increases $F(p)$, and thus decreases $\alpha_V(d)$. We therefore increase d to d_{SC} , which satisfies

$\alpha_V(d_{SC}) = e^* \alpha \Leftrightarrow \int_0^{d_{SC}} \int_0^{\pi_A} (e^* p - p) d\hat{\Omega} + \int_{d_{SC}}^{e^*} \int_0^{\pi_A} (e^* p - d_{SC}) d\hat{\Omega} = \int_0^{\bar{p}} \int_0^{\pi_A} (u_V(e^*) - e^* u(e^*)) d\hat{\Omega}$. The value of d_{SC} also needs to satisfy $d_{SC} < p'$. It is therefore possible to implement an efficient contract using SP whenever

$$\begin{aligned} & \int_0^{p'} \int_0^{\pi_A} (e^* p - p) \hat{\omega}(p, \pi) d\pi dp + \int_{p'}^{e^*} \int_0^{\pi_A} (e^* p - p') \hat{\omega}(p, \pi) d\pi dp \\ & \geq \int_0^{\bar{p}} \int_0^{\pi_A} (u_V(e^*) - e^* u(e^*)) \hat{\omega}(p, \pi) d\pi dp. \end{aligned} \quad (A.1)$$

Note that the first term on the left-hand side is positive (since $\hat{\omega}(p, \pi) < 0$ for $p < p'$), whereas the second is negative (since $\hat{\omega}(p, \pi) > 0$ for $p > p'$). The condition for SP to implement the efficient contract is thus that this first term is sufficiently large to satisfy condition A1. This requires that $\hat{\omega}(p, \pi)$ is sufficiently negative (i.e., $\bar{\omega}(p, \pi)/\underline{\omega}(p, \pi)$ sufficiently low) for low values of p and π (namely for $p \in (0, p')$ and $\pi \in (0, \pi_A)$). Intuitively, this means that the less favorable distribution puts sufficiently more weight on low realizations of p and π , relative to the more favorable distribution. If condition (A.1) is satisfied, $U_V = \bar{u}_V(d_{SC}) - \bar{s}_V^2/2 - x$. We thus have $x_{SC} = \bar{u}_V(d_{SC}) - \bar{s}_V^2/2$. For $x < x_{SC}$, it is possible to use SC to implement the efficient outcome.

Next I show how PC can be used to implement an efficient contracting for an extended range of values of x . Consider a PC contract with $d_{PC} = d_{SC}$ and $c > p'$, such that α_V^A is the same as before. We can find the appropriate value of c as follows. Set c so that $\alpha_V^A(d_{PC}, c) = \int_0^{\bar{p}} \int_0^{\pi_A} F_{PC} d\hat{\Omega} = \int_0^{\bar{p}} \int_0^{\pi_A} F_{SC} d\hat{\Omega} = \alpha_V^A(d_{SC})$. Note that $F_{PC} - F_{SC} = 0$ for $p < d_{SC}$, $F_{PC} - F_{SC} = d + e^*(p - d) - \text{Max}[d, e^*p] > 0$ for $d_{SC} < p < c$, and $F_{PC} - F_{SC} = 0$ for $p > c$, so that $F_{PC} - F_{SC}$ is nonnegative. For $p < p'$ we have $\int_0^{p'} \int_0^{\pi_A} (F_{PC} - F_{SC}) d\hat{\Omega} < 0$ and for $p > p'$ we have $\int_{p'}^c \int_0^{\pi_A} (F_{PC} - F_{SC}) d\hat{\Omega} > 0$. The appropriate value of c is thus given by $\int_0^{p'} \int_0^{\pi_A} (F_{PC} - F_{SC}) d\hat{\Omega} + \int_{p'}^c \int_0^{\pi_A} (F_{PC} - F_{SC}) d\hat{\Omega} = 0$. The condition for such a c to exist is that at $c = \bar{p}$,

$$\int_0^{p'} \int_0^{\pi_A} (F_{PC} - F_{SC}) \hat{\omega}(p, \pi) d\pi dp + \int_{p'}^{\bar{p}} \int_0^{\pi_A} (F_{PC} - F_{SC}) \hat{\omega}(p, \pi) d\pi dp \geq 0. \quad (A.2)$$

This requires that $\widehat{\omega}(p, \pi)$ is sufficient large (i.e., $\overline{\omega}(p, \pi)/\underline{\omega}(p, \pi)$ sufficiently high) for large values of p and small values of π (namely for $p \in (p', \bar{p})$ and $\pi \in (0, \hat{\pi}_A)$). Condition (A.2) is closely related to but different than condition (A.1). Intuitively, condition (A.2) requires that the more favorable distribution puts sufficiently more weight on high realizations of p , relative to the less favorable distribution. By construction, PC has the same incentive power as SC. Moreover, since $F_{PC} \geq F_{SC}$ (with a strict inequality for some p), $\tilde{u}_V^A(d_{PC}, c) > \tilde{u}_V^A(d_{SC})$. This shows that V gets a higher utility from PC than from SC. So far $d_{PC} = d_{SC}$. To further expand the range of PC, we can increase d_{PC} . Consider varying c so as to keep α_V^A constant. A higher d_{PC} increases $\underline{u}_V^A$, and since $\bar{u}_V^A = \alpha_V^A + \underline{u}_V^A$, it also increases $\tilde{u}_V^A(d_{PC}, c)$. Using $d_{PC} = p'$, x_{PC} is obtained from $x_{PC} = \tilde{u}_V(d_{PC} = p', c) - \tilde{s}_V^2/2$. Note also that raising d above p' would disturb the balance of early incentives. For $x > x_{PC}$ it would become necessary to trade off early versus late incentive inefficiencies. Such an analysis is no longer tractable. That is why, as mentioned in the main text, I limit the analysis to the region of efficient contracting.

So far we have assumed $\tilde{\theta}_E = \theta_E$ and $\tilde{\theta}_V = \theta_V$. It is easy to see how the proof generalizes. Clearly, only relative productivity matters. For $\tilde{\theta}_E/\tilde{\theta}_V > \theta_E/\theta_V$, it is efficient to give E even stronger early incentives. In this case, the need to give V additional cash flows on the downside is even more pronounced, and the proof is exactly the same. For $\tilde{\theta}_E/\tilde{\theta}_V < \theta_E/\theta_V$, but $\tilde{\theta}_E/\tilde{\theta}_V$ sufficiently close to θ_E/θ_V , continuity ensures that the proof is also the same. Finally, if $\tilde{\theta}_E/\tilde{\theta}_V$ is considerably smaller than θ_E/θ_V , then it is possible that pure equity gives V too few early incentives. In this case, SC is not optimal, but PC is. To see this, note that we can construct a PC with $c > p'$, so that the PC gives the same incentives as pure equity. This gives V higher cash flow rights and the same incentives. We then decrease d_{PC} and/or increase c , so as to increase V 's incentives. The optimal contract again satisfies $\alpha_V(d_{PC}, c) = \alpha_V^*$, which ensures that V has efficient incentives.

Appendix A.5. Discussion of minimum price requirements

Suppose a venture capitalist invests an amount X . The number of preferred shares he receives are denoted by N_V . Let N be the total number of shares, so that $N_V = eN$. The price per share is given by $P_V = X/N_V$. Suppose the company goes public after t years at a price P_I , implying an IPO value of $i = P_I N$. A minimum price requirement specifies that conversion of preferred shares is automatic only if P_I is greater than some pre-specified threshold P_μ . For $P_I < P_\mu$, the minimum price requirement is binding, implying that automatic conversion does not apply.

Suppose for a moment that the venture capitalist wants to cash out at the IPO. He can either redeem his preferred claim or convert into common equity. For PC, conversion is never optimal. For SC, conversion is optimal whenever $P_I N_V > d \Leftrightarrow P_I > d/N_V \equiv P_\psi$. Let us call P_ψ the conversion price. If $P_\mu < P_\psi$, then there clearly exists a range of IPO prices $P_I \in (P_\mu, P_\psi)$ where automatic conversion matters. However, if $P_\mu > P_\psi$, then it appears that, at least for SC, the minimum price requirement makes automatic conversion redundant. However, this assumes that the investor always wants to convert for $P_I > P_\psi$. I now show that investors actually prefer to delay the conversion decision.

Consider now what happens when the minimum requirement is binding, i.e., when there is no automatic conversion. At the time of the IPO, the venture capitalist could cash out. However, in the absence of any immediate liquidity needs, this would be a suboptimal

strategy. The venture capitalist is better off holding the security (or selling it after the lock-up period), because the preferred terms constitute a put option, where the venture capitalist has the option to redeem his preferential cash flow claims in case of a sufficiently low stock price. An additional reason not to cash out at the IPO is that it can be a negative signal to uninformed stock market investors (see Grinblatt and Hwang, 1989; Habib and Ljungqvist, 2001). After the IPO, the stock price follows some stochastic process, the exact specification of which does not matter here. All that matters is that there is a possibility that the stock price falls below the conversion price P_ψ . The preferred security is more valuable than common equity, because it includes the option value of the preferred terms. It follows that voluntary conversion at the time of the IPO is never an optimal strategy, since it means forgoing this option value. This argument is similar to Steins' (1992) analysis of convertible bonds. Note also that preferred equity typically has redemption rights, which allow the investor to put the claim to the company. But even without redemption rights, preferred equity is more valuable than common equity, because it has additional cash flow rights in all liquidation events where the price falls below the conversion price P_ψ .

If the venture capitalist holds on to his preferred shares, his cash flow rights at the time of the IPO amount to $\Phi(i) = ei + o(i)$, where $o(i) > 0$ denotes the option value that is inherent in the preferred terms. In general, $o(i)$ depends on the stochastic price process. The analysis here does not require a precise functional form of o . For simplicity, assume that the IPO value i itself does not depend on whether the preferred shares are converted or not.

Understanding that preferred terms constitute a put option clarifies the role of the conversion price P_ψ : it represents the strike price of the put option. The option is clearly valuable for $P_I < P_\psi$, where it is “in the money” (i.e., the preferred terms are worth more than conversion). For $P_I > P_\psi$, it is “out of the money.” But the put option remains valuable, since the stock price is stochastic, and could fall below P_ψ at some future point in time.

The first main insight is that if the venture capitalist always had to cash out at the IPO, then it would be true that setting P_μ above P_ψ would make the automatic conversion clause redundant. However, if the preferred terms constitute a put option, automatic conversion matters for all IPO prices, because it eliminates that put option.

The second main insight from this analysis is that minimum price requirements generate a concavity in the venture capitalist's payoff function. The effective share of the venture capitalist is given by e for $P_I > P_\mu$, but $e + o(i)/i$ for $P_I < P_\mu$. Even within the range $P_I < P_\mu$, the venture capitalist's payoff is concave. To see this, note that a sufficient condition for $o(i)/i$ to be decreasing in i is that $o(i)$ is decreasing in i . This follows from the fact that the higher is P_I , the smaller is the probability that the price will fall below P_ψ .

References

- Aghion, P., Bolton, P., 1992. An incomplete contracts approach to financial contracting. *Review of Economic Studies* 59, 473–494.
- Aghion, P., Bolton, P., Tirole, J., 2004. Exit options in corporate finance: liquidity versus incentives. *Review of Finance* 8, 327–353.
- Bascha, A., Walz, U., 2001. Convertible securities and optimal exit decisions in venture capital finance. *Journal of Corporate Finance* 7, 285–306.
- Bergemann, D., Hege, U., 1998. Venture capital financing, moral hazard, and learning. *Journal of Banking and Finance* 22, 703–735.

- Berglöf, E., 1994. A control theory of venture capital finance. *Journal of Law, Economics and Organization* 10, 247–267.
- Black, B.S., Gilson, R.J., 1997. Venture capital and the structure of capital markets: banks versus stock markets. *Journal of Financial Economics* 47, 243–277.
- Casamatta, C., 2003. Financing and advising: optimal financial contracts with venture capitalists. *Journal of Finance* 58, 2059–2086.
- Cestone, G., 2001. Venture capital meets contract theory: risky claims or formal control? CEPR Discussion Paper 3462.
- Cestone, G., White, L., 2003. Anti-competitive financial contracting: the design of financial claims. *Journal of Finance* 58, 2109–2141.
- Cornelli, F., Yosha, O., 2003. Stage financing and the role of convertible debt. *Review of Economic Studies* 70, 1–32.
- Dessein, W., 2005. Information and control in alliances & ventures. *Journal of Finance* 60, 2513–2549.
- Dessi, R., 2005. Start-up finance, monitoring and collusion. *The Rand Journal of Economics* 36, 255–274.
- Fenwick, West, L.L.P., 2003, 2004. Trends in Legal Terms in Venture Financing. www.fenwick.com.
- Gilson, R., Schizer, D., 2003. Understanding venture capital structure: a tax explanation for convertible preferred stock. *Harvard Law Review* 116, 874–899.
- Gompers, P., 1997. Ownership and control in entrepreneurial firms: an examination of convertible securities in venture capital. Mimeo, Harvard Business School.
- Green, R., 1984. Investment incentives, debt and warrants. *Journal of Financial Economics* 13, 115–136.
- Grinblatt, M., Hwang, C., 1989. Signaling and the pricing of new issues. *Journal of Finance* 44, 393–420.
- Habib, M., Ljungqvist, A., 2001. Underpricing and entrepreneurial wealth losses in IPOs: theory and evidence. *Review of Financial Studies* 14, 433–458.
- Hart, O., Moore, J., 1998. Default and renegotiation: a dynamic model of debt. *Quarterly Journal of Economics* 113, 1–41.
- Hellmann, T., 1998. The allocation of control rights in venture capital contracts. *Rand Journal of Economics* 29, 57–76.
- Hellmann, T., 2002. A theory of strategic venture investing. *Journal of Financial Economics* 64, 285–314.
- Hellmann, T., 2003. Going public and the option value of convertible securities in venture capital. In: McCahery, J., Renneboog, L. (Eds.), *Venture Capital Contracting and the Valuation of High Tech Firms*. Oxford University Press, Oxford, pp. 60–73.
- Hellmann, T., Puri, M., 2000. The interaction between product market and financing strategy: the role of venture capital. *Review of Financial Studies* 13, 959–984.
- Hellmann, T., Puri, M., 2002. Venture capital and the professionalization of start-up firms: empirical evidence. *Journal of Finance* 57, 169–197.
- Hsu, D., 2004. What do Entrepreneurs pay for venture capital affiliation? *Journal of Finance* 59, 1805–1844.
- Inderst, R., Müller, H., 2004. The effect of capital market characteristics on the value of start-up firms. *Journal of Financial Economics* 72, 319–356.
- Kaplan, S., Strömberg, P., 2001. Venture capitalists as principals: contracting screening and monitoring. *American Economic Review Papers and Proceedings* 91, 426–430.
- Kaplan, S., Strömberg, P., 2003. Financial contracting meets the real world: an empirical study of venture capital contracts. *Review of Economic Studies* 70, 281–315.
- Kaplan, S., Strömberg, P., 2004. Contracts, characteristics, and actions: evidence from venture capitalist analyses. *Journal of Finance* 59, 2177–2210.
- Kirilenko, A., 2001. Valuation and control in venture finance. *Journal of Finance* 56, 565–587.
- Marx, L., 1998. Efficient venture capital financing combining debt and equity. *Review of Economic Design* 3, 371–387.
- Moore, J., Repullo, R., 1988. Subgame perfect implementation. *Econometrica* 56, 1191–1220.
- Repullo, R., Suarez, J., 2004. Venture capital finance: a security design approach. *Review of Finance* 8, 75–108.
- Sahlman, W., 1990. The structure and governance of venture capital organizations. *Journal of Financial Economics* 27, 473–521.
- Schmidt, K., 2003. Convertible securities and venture capital finance. *Journal of Finance* 58, 1139–1166.
- Stein, J., 1992. Convertible bonds as backdoor equity financing. *Journal of Financial Economics* 32, 3–21.
- Ueda, M., 2004. Banks versus venture capital: project evaluation, screening, and expropriation. *Journal of Finance* 59, 601–621.